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Wage inequality, technology, and trade

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Abstract

The recent widening of wage inequality has been attributed by some to skill-biased-technical-change and by others to trade liberalization. This paper examines the two explanations within a unified model and also presents a new modeling of skill-biased-technical-change, where skilled workers replace unskilled ones. As a result technology adoption is endogenous and does not occur in all countries. Hence, wages for both types of workers, trade patterns and also factor productivities in all countries are endogenously determined. The model sheds light on the relationship between technology and trade, on the reasons for global productivity differences and on the causes for the recent rise in wage inequality.

JEL classification: D3; F16; J31

Keywords: Wage inequality; Skill-biased technical change; International trade; Productivity differences

1. Introduction

In recent decades we have seen a dramatic rise in wage inequality in the US. Similar, though smaller, changes have been observed in other countries as well.[1] A number of explanations have been offered to this rise in wage inequality. The most popular explanations are skill-biased-technical-change on the one hand and the liberalization of international trade on the other hand.[2]

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[1] For evidence on this development see Davis and Haltwinger [12], Katz and Murphy [18], Juhn et al. [17], Davis [11] and Berman et al. [7].

[2] Bound and Johnson [8], Katz and Murphy [18], Greenwood and Yorukoglu [15], Acemoglu [1], Galor and Moav [13] and many others stress the role of skill-biased-technical-change. Leamer [19], Wood [21], Hanson and Harrison [16] and others focus on the role of trade liberalization. There have been other explanations to the rise in wage inequality, such as reduction in education supply, as suggested by Goldin and Katz [14] and Card and Lemieux [9].

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This paper presents two theoretical contributions to this area. One is to embed the two explanations together within a unified model of trade and technology. [3] The second contribution is a new way to model skill-biased-technical-change, as innovations that enable replacing unskilled workers by skilled ones. The novelty of this model is that such innovations are not everywhere adopted, but only where the wage rates induce adoption. Hence, wages, trade patterns, and technology adoption are all jointly determined in this model. Interestingly, differences across countries in technology adoption also lead to differences in factor productivity. As a result the paper contains contributions also to the literature on productivity differences. [4]

The paper presents a model in which the final good is produced by many intermediate goods. There exist primitive technologies that enable production of all intermediate goods by unskilled workers. New innovations enable producers to replace unskilled workers in production of some intermediate goods by fewer but skilled workers. Hence, technical progress replaces one input by another. This has two results. The first is that technology adoption increases demand for skilled workers and reduces demand for unskilled ones, so that the wage gap between the two types of workers increases. The second result is that such innovations are not everywhere adopted by producers, as adoption depends on input prices, namely on the wage ratio between skilled and unskilled workers. [5] Hence technology adoption differs across countries.

This result leads not only to endogenous technology adoption, but also to endogenous determination of trade patterns. Countries with many skilled workers adopt all new technologies and are called developed, while countries with fewer skilled workers do not adopt all available technologies and are called less developed. Hence, countries specialize in different intermediate goods, thus leading to international trade. We assume that only some intermediate goods are tradable and trade liberalization is modeled as increasing the set of tradable goods. While technical progress and trade liberalization are assumed to be independent, the patterns of trade are clearly endogenous and are affected by technical progress.

The model has many results. While some are already known, some results are new and surprising. The more standard results are that in developed countries both technical progress and trade liberalization increase wage inequality, while in less developed countries technical progress increases wage inequality but trade liberalization reduces it. What happens to the patterns of trade between developed and less developed countries is more surprising. While trade liberalization increases the share of trade in income in both countries, technical progress increases the share of trade in income only in the developed countries, while it reduces this share in less developed countries. This surprising prediction of the model is contrasted with some data and according to these, trade between developed and less developed countries, measured as shares in GDP, did not change much over the last 20 years. This might suggest that the effect of trade liberalization on the recent rise in wage inequality has been rather small.

Another interesting set of results refers to the emergence of productivity difference between countries. These differences are a result of differences in human capital, but are also amplified by endogenous technology adoption. Thus, countries with more skilled workers produce more intermediate goods by use of skilled technologies, which are more labor saving. Hence such countries have higher productivity. Another interesting result is that international trade

[3] Acemoglu [2] also combines technology and trade in explaining the skill premium, but in a very different way, as discussed below.

[4] This recent literature contains theoretical contributions, like Acemoglu and Zilibotti [3] and much empirical research, called 'development accounting', which is surveyed and assessed by Caselli [10].

[5] A similar result appears also in Zeira [22], but in a different context with labor and capital.

can amplify productivity differences across countries, for a reasonable set of parameters. This is explained as follows. Trade leads to specialization in skill in developed countries and as skilled production is more labor-saving than unskilled production, the gap between countries increases. Thus, this model can also contribute to the explanation of TFP differences across countries.

As mentioned above, Acemoglu [2] also presents a unified framework where trade liberalization and technical change affect wages. His paper though is very different from this one, in many ways. First, its main point is that trade liberalization itself induces skill-biased-technical-change, since it raises prices of skilled goods. Once invented, these new technologies are adopted everywhere. This paper instead focuses on adoption of technology rather than on its creation. It presents a framework where technologies are not adopted everywhere, and adoption depends on factor prices. Hence, the developed countries adopt more technologies, while the less developed countries adopt less. Furthermore, Acemoglu [2] does not examine at all the effect of technical change on trade patterns, which is one of the main issues in this paper, and it also does not study the effect of trade on the income gap between developed and less developed countries.

The paper is organized as follows. Section 2 presents the model, while Section 3 describes the equilibrium in closed economies. Section 4 analyzes the effects of technical progress and human capital acquisition on wages, technology adoption and productivity. Section 5 describes the equilibrium with international trade. Section 6 examines the effects of technical progress and trade liberalization on the global economy. Section 7 analyzes productivity differences in the model and Section 8 concludes. Appendix A contains proofs.

2. The model

Consider a world with one final good, which is used for consumption only. The final good Y is produced by a continuum of intermediate goods $i \in [0, 1]$. Production of the final good is described by the following Cobb–Douglas production function:

$$\log Y = \int_0^1 \log X(i)\, di, \tag{1}$$

where Y is the amount of the final good, and $X(i)$ is the amount of intermediate good i. [6]

Intermediate goods are produced in two alternative technologies, one by unskilled labor and the other by skilled labor, both with fixed marginal productivities. Productivity depends on technology, skilled or unskilled, on the intermediate good, and on the country of production. Thus, production of one unit of intermediate good i with the unskilled labor technology in country j requires $n(i)/a_j$ units of unskilled labor. Production of one unit of intermediate good i with the skilled labor technology in country j requires $s(i)/a_j$ units of skilled labor. Note, that country productivity a_j is assumed to be the same for all intermediate goods, whether they are produced by skilled or by unskilled labor. This productivity therefore reflects general country characteristics, like geography, infrastructure, institutions, rule of law, etc.

We assume that: $s(i) < n(i)$, namely that the skilled labor technology enables reduction of labor input, but it requires a different type of labor input. Hence, the benefit of this technology, of reducing labor input, comes at a cost, of increasing the skill input. [7] Denote the relative gain

[6] The results carry through to other production functions, like CES, as well.

[7] A similar modeling of innovations that require replacing labor by capital appears in Zeira [22].

in labor from replacing unskilled workers by skilled workers, by $g(i)$:

$$g(i) = \frac{n(i)}{s(i)} > 1. \tag{2}$$

Note that the function g is independent of the country of production. It is assumed to be a decreasing function, namely intermediate goods are ordered by decreasing relative labor gain in replacing unskilled by skilled. Furthermore, g is assumed to be continuous.

While the unskilled labor technologies are known from time immemorial, the skilled labor technologies are not known for all intermediate goods and are invented over time. At period t these technologies are known for only some intermediate goods, i.e. for a set $F_t \subset [0, 1]$. Technical progress means increasing F_t over time, thus enabling to replace unskilled labor by skilled in more intermediate goods, namely: $F_t \supseteq F_{t-1}$, for all t. In this sense technical progress in this model is skilled biased. Note that F_t contains all available technologies in time t, but they are not automatically adopted.

We further specify the set of skilled technologies by assuming that the most rewarding technologies are invented first. Namely, technologies with higher relative gains, i.e. with higher g, are researched and invented earlier. Hence:

$$F_t = [0, f_t]. \tag{3}$$

The variable f, which is called the technology frontier, therefore measures the level of technical progress. Initially it is assumed that f is exogenously determined. Section 4.3 endogenizes technical progress by assuming that creation of new technologies is costless. It then shows that indeed the technologies with the highest gains are invented, as assumed by Eq. (3).

Next we describe labor supplies of skilled and unskilled workers. Since the analysis focuses on the short and medium run, we do not specify the process of skill acquisition and assume that in each period the supplies of skilled and unskilled workers are given. Denote the size of the labor force in the country be L, and assume that a share h of it be skilled and a share $1 - h$ is unskilled. Each worker supplies one unit of labor in a period of time. Hence, supplies of both types of labor are perfectly inelastic.

Markets are assumed to be perfectly competitive. It is also assumed that the final good is not traded, but some of the intermediate goods are traded. More specifically, the set of traded intermediate goods is M_t, which is uniformly distributed over [0, 1]. This set is determined by type of goods, by geography and by policy. The size of the set M_t, namely the amount of traded goods, is a measure for trade openness:

$$m_t = \int_{M_t} di. \tag{4}$$

To focus attention on the aggregate effects of developed and less-developed countries, we consider a global model of two countries only, A and B. The two countries differ in productivity, A having higher productivity than B, namely: $a_{\mathrm{A}} > a_{\mathrm{B}}$. We also assume that country A has relatively more skilled workers than country B: $h_{\mathrm{A}} > h_{\mathrm{B}}$. The two countries differ in population as well, namely by the sizes of labor forces, L_{A} and L_{B}, respectively.

3. Equilibrium in a country without trade

We consider first an economy, which is closed to international trade, namely the set M of tradable intermediate goods is null. Technologies, though, are available in all countries. Since the following analysis fits any country, we delete country subscript in this and in the following section. Denote the price of intermediate good i by $p(i)$, the wage of skilled workers by w_s, and

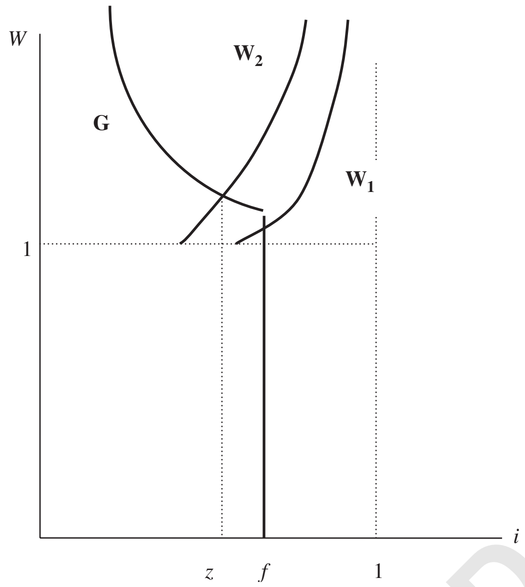

Fig. 1.

the wage of unskilled workers by w_n.[8] Since the economy is closed, these prices are determined domestically.

Let us first describe the choice of technology in the production of each intermediate good. If the skilled technology for intermediate good i has not been invented yet, i.e. if $i > f_t$, then clearly the unskilled technology is used. If the skilled technology has already been invented, namely if $i \leqslant f_t$, producers can choose between the two technologies. They adopt the new technology, which uses skilled labor, if

$$s(i)w_s/a \leqslant n(i)w_n/a,$$

or if:

$$g(i) \geqslant \frac{w_s}{w_n}. \tag{5}$$

Hence, producers of intermediate goods with the highest gain from skill adopt the new technologies and hire skilled workers. Note, that the wage ratio between skilled and unskilled workers, which we denote by W and call the 'wage ratio', determines the level of technology adoption. This variable describes wage inequality and is a central variable in our analysis. The set of adopted technologies Z is, therefore, equal to $Z = [0, z]$, where z is determined by

$$z = \min\{g^{-1}(W), f\}. \tag{6}$$

The highest technology adopted z is described by the curve **G** in Fig. 1 below.

We can, therefore, distinguish between two cases. In the first case the wage ratio is high enough to deter some technology adoption, so that not all available technologies are adopted. In the second

[8] From here on we delete time subscripts wherever possible.

case the wage ratio is lower and all available skilled technologies are adopted. We call a country that adopts all skilled technologies $[0, f]$ 'developed' and a country that adopts only some of these technologies 'less developed'.

To close the equilibrium we show how the wage ratio is determined. We begin with the goods markets to describe price determination. On the demand side for intermediate goods we get the following first-order condition for each i:

$$p(i) = \frac{\partial Y}{\partial X(i)} = \frac{Y}{X(i)}. \tag{7}$$

On the supply side of intermediate goods constant marginal productivity and zero profits lead to:

$$p(i) = \begin{cases} w_s s(i)/a & \text{if } i \in Z, \\ w_n n(i)/a & \text{otherwise.} \end{cases} \tag{8}$$

Wages are determined by the equilibrium conditions in the two labor markets, for skilled and unskilled labor. Equilibrium in the market for skilled labor is reached when supply equals demand. Together with (7) and (8) we get:

$$Lh = \int_0^z [s(i)/a]X(i)\,di = z\frac{Y}{w_s}. \tag{9}$$

Similarly, the equilibrium condition in the market for unskilled labor is

$$L(1-h) = \int_z^1 [n(i)/a]X(i)\,di = (1-z)\frac{Y}{w_n}. \tag{10}$$

From (9) and (10) we get:

$$W = \frac{w_s}{w_n} = \frac{1-h}{h}\frac{z}{1-z}. \tag{11}$$

The right-hand side of (11), which describes the wage ratio, is an increasing function of z. It is described by the curve **W** in Fig. 1. Note that the curve is drawn only for $W \geqslant 1$, since skilled workers can always work as unskilled, if wages for unskilled labor are higher than wages of skilled. Then the wage ratio does not fall below 1 and h adjust itself so that (11) still holds. Hence, the **W** curve is horizontal at 1, while above it, it is determined by Eq. (11) with fixed supplies of skilled and unskilled workers, namely a fixed h.

The equilibrium wage ratio and the level of technology adoption are jointly determined by the intersection of the **W** and **G** curves in Fig. 1. There are two cases. If relative supply of skilled workers is large enough, as in curve $\mathbf{W_1}$, then $z = f$, all new innovations are adopted and the economy is developed. If the relative supply of skilled workers is low, as in curve $\mathbf{W_2}$, some of the new innovations, between z and f, are not adopted and the economy is less developed. In this case technology adoption z and the wage ratio are determined by

$$\frac{1-h}{h} \cdot \frac{z}{1-z} = g(z). \tag{12}$$

For simplicity and realism it is assumed that the equilibrium wage ratio W is always greater than 1, namely that: $f > h$.

To complete the description of equilibrium we derive the absolute wage levels of skilled and unskilled workers. Substituting the first-order conditions (7) in the production function (1) leads to

$$\int_0^1 \log p(i)\, di = 0. \tag{13}$$

By substituting (8) and the wage ratio (11) into (13) we can calculate the two wage levels. The wage of unskilled workers is

$$\log w_n = \log a - z \log \frac{z}{1-z} - z \log \frac{1-h}{h} - \int_0^z \log s(i)\, di - \int_z^1 \log n(i)\, di. \tag{14}$$

The wage of skilled workers is

$$\log w_s = \log a + (1-z) \log \frac{z}{1-z} + (1-z) \log \frac{1-h}{h} - \int_0^z \log s(i)\, di - \int_z^1 \log n(i)\, di. \tag{15}$$

Note that if the country is developed and $z = f$, the variables h and f are independent. But if the country is less developed, human capital determines the level of technology adoption z, as shown in Fig. 1 and in Eq. (12). In that case we can rewrite the wages of skilled and unskilled in terms of z only as follows:

$$\log w_n = \log a - z \log g(z) - \int_0^z \log s(i)\, di - \int_z^1 \log n(i)\, di. \tag{16}$$

And

$$\log w_s = \log a + (1-z) \log g(z) - \int_0^z \log s(i)\, di - \int_z^1 \log n(i)\, di. \tag{17}$$

In the next section we analyze this equilibrium and examine how wage inequality, technology, human capital and productivity are linked together.

4. Discussion of the closed economy

4.1. Effects of technical progress and of education

We first analyze the effect of skill-biased-technical-change on the closed economy. An increase in f shifts the vertical part of the **G** curve in Fig. 1 to the right. If the country is developed the wage ratio W rises and z increases, namely more technologies are adopted. The skilled wage rises, as seen by derivation of (15):

$$\frac{\partial \log w_s}{\partial f} = \frac{1}{f} + \log g(f) - \log W.$$

Furthermore, the elasticity of skilled wages with respect to technical progress is greater than 1. The effect of technical change on unskilled wages is ambiguous:

$$\frac{\partial \log w_n}{\partial f} = -\frac{1}{1-f} + \log g(f) - \log W.$$

If the technology frontier is close to the intersection of **G** and **W**, namely if W is close to $g(f)$, then unskilled wages might even fall as a result of technical progress.

If the country is less developed, the equilibrium is not affected at all by the skill-biased-technical-change: the wage ratio W, technology adoption $\underline{z}$, and the wages of skilled and unskilled do not change. Hence, a developed economy reacts to skill-biased technical change by more than a less developed country. [9]

Another exogenous change that can affect the closed economy is investment in human capital, namely an increase in h. Such a change shifts the **W** curve down and reduces the wage ratio. Note that the effect on the wage ratio is smaller in a less developed economy than in a developed one, as shown in Fig. 1. This is since in the less developed country the increase in skill leads to adoption of more skilled technologies. This increases the demand for skilled workers and reduces the demand for unskilled, which mitigates the reduction of W. Analysis of Eqs. (14)–(17) shows that w_n rises with h and w_s falls with h.

4.2. Productivity or TFP

In this sub-section we calculate total factor productivity in the closed economy and examine how it is affected by technical progress and by investment in human capital, namely by increasing f or h, respectively. It should be noted that this is a simplified model of TFP, as it does not include physical capital. But even this simplified model has interesting insights, and shows that the model of technology adoption that requires changes in inputs can be very useful for the study of technical change. Some of the following results are compared with the findings of Caselli [10] and of Acemoglu and Zilibotti [3] on productivity differences across countries.

The standard definition of TFP in a model without capital is labor productivity. Output per worker in units of the final good in this model is equal to:

$$TFP = \frac{Y}{L} = w_s h + w_n(1-h) = w_n[1 + h(W-1)]. \tag{18}$$

From Eq. (18) it is clear that TFP is affected by human capital, both directly and indirectly, and by technical progress, if the economy is developed. First, it can be shown that the effect of technical progress on productivity of a developed economy is positive:

$$\frac{\partial \log TFP_D}{\partial f} = \log g(f) - \log W \geqslant 0. \tag{19}$$

Since technical progress has no effect on a less developed country it follows that technical progress increases the productivity difference between developed and less developed countries. Note that Acemoglu and Zilibotti [3] provide a very different explanation to such productivity differences. They claim that technical change is skill-biased as it fits the needs of the North, while in the South there is mismatch between such technologies and the skill supply. In this model instead technical progress is not adopted at all in the South, as wages of unskilled are much lower than wages of skilled and so producers have no incentive to use a new technology that replaces unskilled by skilled.

Eq. (18) also enables us to analyze the effect of human capital acquisition on TFP, which can be divided to three separate effects. The first is the direct positive effect through the increase in skill. This is shown in Eq. (18) by an increase in h. The second effect is that an increase in supply

[9] Interestingly, the more common modeling of skill-biased-technical-change, as an increase in the productivity of skilled workers in all jobs, namely an upward shift of the function g, yields an opposite result. The wage ratio in a less developed country rises, while in a developed country it remains unchanged.

of skill reduces the wage ratio and raises wages of unskilled. This effect works in both developed and less developed countries. The third effect is that in a less developed country an increase in human capital increases technology adoption and thus raises TFP. To get an idea of the relative size of the three effects note first that (18) can be written as

$$TFP = w_n \frac{1-h}{1-z}. \tag{20}$$

By use of (20) and (14) we get:

$$\frac{d \log TFP_D}{dh} = \frac{f-h}{h(1-h)} = \frac{1-f}{1-h}(W-1).$$

Hence, the sum of the first and second effects is positive, due to the assumption that $W > 1$. The third effect, of greater technology adoption, which is experienced in less developed countries only, increases TFP by even more. Hence, the effect of human capital acquisition is stronger for less developed countries than for developed ones. The following lemma shows that the third and second effects cancel each other in the less developed economy and the indirect effect of human capital acquisition is equal to zero.

Lemma 1. *The overall effect of human capital acquisition on the less developed country is equal to the direct effect of human capital acquisition*:

$$\frac{d \log TFP_{\mathrm{LDC}}}{dh} = \frac{\partial \log TFP_{\mathrm{LDC}}}{\partial h} = \frac{z-h}{h(1-h)} = \frac{1-z}{1-h}(W-1).$$

Proof. In Appendix A. □

Many recent empirical studies have attempted to measure differences in productivity across countries after controlling for human capital or skill. This literature is summarized in the recent exhaustive and authoritative survey by Caselli [10]. The way these studies control for human capital is by deducting from TFP a measure of the average level of skill, weighted by decreasing gains from education. This measure is exactly what we denote in Eq. (18):

$$1 + h(W-1).$$

Note, that by deducting this measure of human capital we are left with w_n, which is what these studies measure as productivity in addition to human capital. But our model shows that this unskilled wage differs across countries not only because of differences in productivity a, but also because of different levels of human capital, since more human capital induces adoption of technologies, and that increases measured productivity, namely w_n, as shown by Eq. (16). Thus, development accounting does not control for all the effects of human capital.

4.3. *Endogenous technical progress*

In this sub-section we assume that innovations are costlessly created. Hence, an innovation for intermediate good i is created as long as there is a country where this technology will be adopted. Hence, the set of technologies invented is $[0, f]$, since if f is adopted all technologies with higher gains are adopted as well, as shown above. For each country j let z_j be the intermediate good determined by Eq. (12).[10] The country with the highest z_j is the country that determines *f*, the technology frontier. In other words, the country with the highest accumulation of human capital, highest h_j, is also the country that sets the technology frontier. Note that this explanation to why technologies are invented for the developed countries rather than for the less developed, differs from the explanation of Acemoglu and Zilibotti [3]. They assume that only the North respects property rights and thus innovations are tailor-made to the North only. In this model wages of unskilled are higher in the North, and that creates an incentive to invent technologies that replace unskilled workers by skilled workers.

The model with endogenous technology therefore presents a new explanation to skill-biased technical progress. It is a result of investment in human capital in the leading country in the world, namely an increase in h_{A}. That lowers the wage ratio and creates incentives for skill biased technical change, namely an increase in *f*. The new technologies are adopted only in the leading country or region and not in the less developed countries, as shown above. If the change in *f* comes with a lag after h_{A} increases, there is first a decline in W_{A} due to increased skill, and only later a rise in the wage ratio due to skill-biased technical progress. Indeed, these dynamics fit the stylized facts in the US in the 20th century, as described by Goldin and Katz [14]. In the first half of the century the amount of skilled workers significantly increased and the wage ratio fell, while in the second half of the century the wage ratio rose. Note that during all this process TFP rises as well as shown by Lemma 1.[11]

5. World trade equilibrium

In this section we allow for international trade, namely the set *M* of tradable intermediate goods is no longer null and $m > 0$. Denote the wages of skilled workers in the two countries by $w_{s,\mathrm{A}}$ and $w_{s,\mathrm{B}}$ and the wages of unskilled workers by $w_{n,\mathrm{A}}$ and $w_{n,\mathrm{B}}$, respectively. Note that for wages and prices in both countries to be comparable, the numeraire good has to be one of the tradable intermediate goods in *M*. Also, to simplify notation assume that the productivity in country B is 1, and denote the productivity in country A by *a*, namely: $a_{\mathrm{B}} = 1$ and $a_{\mathrm{A}} = a > 1$. Thus, *a* is the a priori productivity ratio between the two countries.

Both technology adoption and the patterns of global trade are determined by the four unit costs of production, of skilled and unskilled production in the two countries. The unit costs of skilled production in A and B are $w_{s,\mathrm{A}}s(i)/a$ and $w_{s,\mathrm{B}}s(i)$, respectively, and the unit costs of unskilled production are $w_{n,\mathrm{A}}n(i)/a$ and $w_{n,\mathrm{B}}n(i)$. We next examine how these costs of production are related to one another by restricting the analysis to the case that A is more developed than B. Namely, focus on the case that A is sufficiently more skill abundant, so that its skilled-unskilled wage ratio is lower than the wage ratio in B: $W_{\mathrm{A}} = w_{s,\mathrm{A}}/w_{n,\mathrm{A}} < W_{\mathrm{B}} = w_{s,\mathrm{B}}/w_{n,\mathrm{B}}$. In this case the following lemma holds.

Lemma 2. *Wages in the two countries satisfy*: $w_{s,\mathrm{A}} \leqslant a w_{s,\mathrm{B}}$ *and* $w_{n,\mathrm{A}} \geqslant a w_{n,\mathrm{B}}$. *One of the inequalities is strict*.

Proof. Note first, that it is impossible that both inequalities are violated, since then the wage ratio in A is higher than in B, which contradicts the above assumption. A similar argument shows that

[10] Note that this z is not bounded by the level of technology as above.

[11] Interestingly, our analysis predicts that the rise of TFP decline in the second half of the period, when skill-biased-technical-change occurs, as W and $g(f)$ become close.

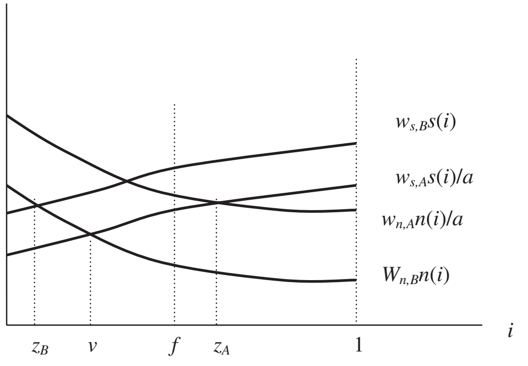

Fig. 2.

the two equalities are impossible. Next note that if only one inequality is violated, for example if $w_{s,\mathrm{A}} > aw_{s,\mathrm{B}}$, we reach a contradiction as well. Consider two possible cases. In the first one $w_{n,\mathrm{A}} > aw_{n,\mathrm{B}}$. In this case country A is importing all goods from country B, since its costs of production of all intermediate goods are higher. This is impossible. In the second case $w_{n,\mathrm{A}} = aw_{n,\mathrm{B}}$. In this case the wage ratio in country A is higher than in country B, which contradicts the above assumption. Hence, both wage inequalities hold. □

Lemma 2 therefore shows that wages of skilled workers in country A cannot exceed wages of skilled in B by more than the productivity factor *a*. The wages of unskilled in country A can exceed the wages of unskilled in B by more, due to the abundance of unskilled workers in B. Lemma 2 also tells us what are the patterns of international trade. Country A is exporting intermediate goods produced by skilled labor, as it produces them at lower costs. Country B is exporting intermediate goods produced by unskilled labor, as it produces them at lower costs than A. Note that if one of the inequalities in Lemma 2 is strict, the respective country produces all the global consumption of the intermediate goods it exports and there is complete specialization. If there is equality, there is some domestic production in the importing country as well. We should therefore distinguish between three cases: full specialization, equality of costs of skilled production in the two countries, and equality of costs of unskilled production in the two countries. We focus the analysis on the first two cases, as the third seems to be less realistic, and it adds no new insight to the analysis.

Fig. 2 describes the four unit costs of production of the two types of technologies in the two countries. It is based on the above assumption that the wage ratio in A is lower than in B and on Lemma 2. Fig. 2 shows both the patterns of technology adoption and of international trade. The case described in Fig. 2 is that of full specialization, but it is easy to imagine the case of equal costs of skilled production, when $w_{s,\mathrm{A}} = aw_{s,\mathrm{B}}$ and the two cost curves for skilled labor in the two countries coincide.

Country B adopts technologies in $[0, z_\mathrm{B}]$, where z_B is the intersection of the curves of unit costs of skilled and unskilled production in B. Country A can adopt all the technologies in $[0, z_\mathrm{A}]$, where z_A is the intersection of the unit costs of skilled and unskilled production in A. Formally: $z_\mathrm{A} = g^{-1}(W_\mathrm{A})$ and $z_\mathrm{B} = g^{-1}(W_\mathrm{B})$. Hence, according to our above assumption $z_\mathrm{A} > z_\mathrm{B}$. Of course, the technologies adopted must first be invented, and Fig. 2 also shows the frontier of technology *f*. Fig. 2 assumes that $f < z_\mathrm{A}$, namely that country A is developed and adopts all available technologies. This is a reasonable assumption since it makes no sense that someone will invent technologies that will be adopted by no one. Hence, technology adoption in A is f and in B is z_{B}.

Fig. 2 also describes the patterns of trade. Country A exports an intermediate good i, which it produces by skilled workers, if its cost is lower not only from skilled production in B, which holds according to Lemma 2, but also from unskilled production in B. Hence, a good $I \in M$ is exported by A if:

$$w_{s,\mathrm{A}} s(i)/a \leqslant w_{n,\mathrm{B}} n(i),$$

or if

$$g(i) \geqslant \frac{1}{a} \frac{w_{s\mathrm{A}}}{w_{n\mathrm{B}}}.$$

Hence, country A is exporting the set of intermediate goods $M \cap [0, f] \cap [0, v]$, where v is determined by the intersection of the relevant curves in Fig. 2 and is described by

$$v = g^{-1}\left(\frac{1}{a} \frac{w_{s\mathrm{A}}}{w_{n\mathrm{B}}}\right). \tag{21}$$

Country B is exporting intermediate goods produced by non-skilled labor, which include all traded goods except those exported by A, i.e. $M \backslash M \cap [0, f] \cap [0, v]$. Note that due to Lemma 1, v is always between z_{A} and z_{B}. In the case of full specialization v is strictly between them, as shown in Fig. 2. If the costs of skilled production in the two countries are equal, v coincides with z_{B}, as is clear from (21) and from Fig. 2. While the analysis that follows assumes that v is smaller than f, if v exceeds f, then f simply replaces v, while the rest of the analysis is unchanged, as shown below.

The amount of intermediate good i in country j, $j \in \{\mathrm{A}, \mathrm{B}\}$, is determined by the first order conditions and is given by

$$X_j(i) = \frac{P_j Y_j}{p_j(i)}. \tag{22}$$

Let P_j be the price of the final good in country j, while $p_j(i)$ is the price of intermediate good i in country j. Note that for traded goods the prices in the two countries must coincide, so: $p_{\mathrm{A}}(i) = p_{\mathrm{B}}(i) = p(i)$.

5.1. *Equilibrium with full specialization*

We first analyze the case of full specialization, where the cost of skilled production in A is strictly lower than in B. In this case B imports from A all traded inputs that are produced by skilled labor and A imports from B the other traded goods, which are produced by non-skilled workers. Since global trade must be balanced we get

$$\int_{M \cap [0,v]} p(i) X_{\mathrm{B}}(i)\, di = \int_{M \cap [v,1]} p(i) X_{\mathrm{A}}(i)\, di. \tag{23}$$

From (22) and (23) we derive the ratio of incomes in the two countries:

$$\frac{P_{\mathrm{B}}\, Y_{\mathrm{B}}}{P_{\mathrm{A}} Y_{\mathrm{A}}} = \frac{1 - v}{v}. \tag{24}$$

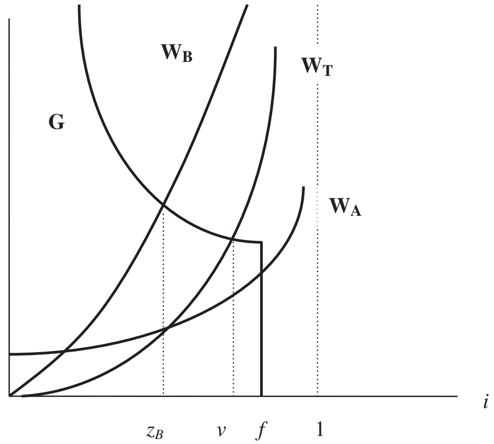

Fig. 3.

Hence, the income ratio between the two countries depends negatively on v. Note that if v exceeds f, then f replaces v in Eq. (24).

The derivation of equilibrium in the global economy follows the four labor market equilibrium conditions and is presented by the following proposition.

Proposition 1. *In the case of full specialization the equilibrium exists and is unique. In this equilibrium the wage ratio in country* A *is*

$$W_{\mathrm{A}} = \frac{1-h_{\mathrm{A}}}{h_{\mathrm{A}}} \frac{1-(1-m)(1-f)}{(1-m)(1-f)}.$$

Technology adoption and the wage ratio in country B *are determined by*

$$W_{\mathrm{B}} = g(z_{\mathrm{B}}) = \frac{1-h_{\mathrm{B}}}{h_{\mathrm{B}}} \frac{z_{\mathrm{B}}(1-m)}{1-z_{\mathrm{B}}(1-m)}.$$

The trade threshold v *is determined by*

$$g(v) = \frac{1}{a} \frac{L_{\mathrm{B}}}{L_{\mathrm{A}}} \frac{1-h_{\mathrm{B}}}{h_{\mathrm{A}}} \frac{1-(1-f)(1-m)}{1-z_{\mathrm{B}}(1-m)} \frac{v}{1-v}.$$

The shares of trade in income in the two countries are mv *in* A *and* $m(1-v)$ *in* B.

Proof. In Appendix A. □

The full specialization equilibrium can be described diagrammatically, as shown in Fig. 3. The **G** curve is the same as in Fig. 1. The curves $\mathbf{W_A}$, $\mathbf{W_B}$ and $\mathbf{W_T}$ describe the right-hand sides of the three conditions in Proposition 1, as functions of f, z_{B} and v, respectively. The intersections with the curve **G** determine the equilibrium values of these variables. Note, that the curve $\mathbf{W_T}$ itself depends on z_{B}. Note also that if there is no trade, namely if $m = 0$, the three curves are proportional to one another. This leads to the following proposition.

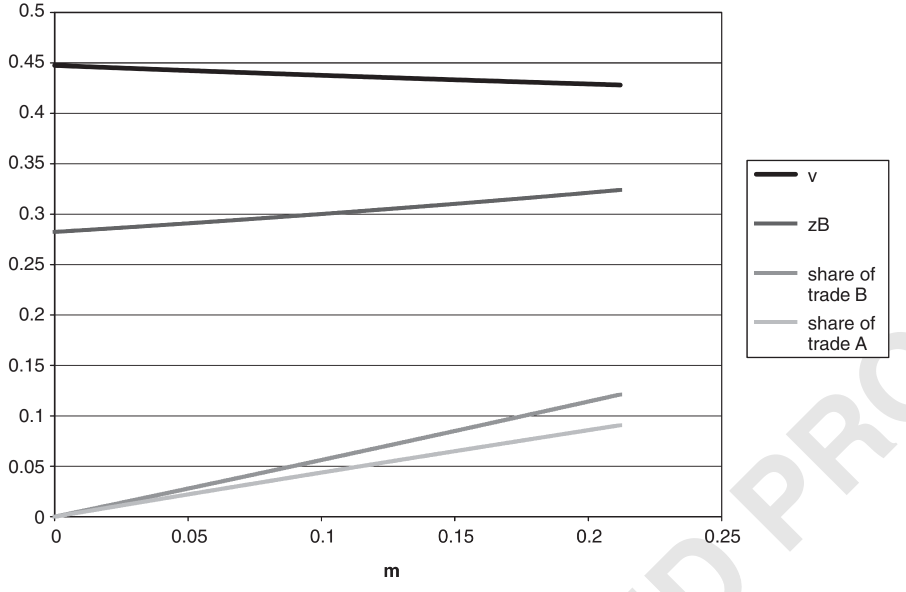

Fig. 4.

Proposition 2. *If technology adoption in* B *in autarky* $\bar{z}_{\mathrm{B}}$ *satisfies*:

$$1 - \bar{z}_{\mathrm{B}} > \frac{f}{a}\frac{h_{\mathrm{B}}L_{\mathrm{B}}}{h_{\mathrm{A}}L_{\mathrm{A}}},$$

then for low levels of m full specialization prevails.

Proof. In Appendix A. □

The condition required by Proposition 2 is fairly reasonable if human capital acquisition in the less developed country is sufficiently low. To get a better idea of how the equilibrium looks like, we present a simulation of a numerical example, which uses some realistic values to the parameters. In this example we assume that the function g is: $g(i) = i^{-1}$. The population ratio is the same as in the world in the 1990s: $L_{\mathrm{B}}/L_{\mathrm{A}} = 6$, and the educational attainments are $h_{\mathrm{A}} = .5$, $h_{\mathrm{B}} = .1$.[12] As for the productivity parameter we choose $a = 3$, based on productivity ratio prior to the industrial revolution.[13] The technology frontier is assumed to be: $f = 0.55$, to calibrate for the current level of wage ratio in the developed countries of around 1.5. Fig. 4 describes the equilibrium as trade is liberalized and m increases from 0 to .21, where country A exactly exhausts the technology frontier. As Fig. 4 shows v exceeds z_{B}, so that full specialization applies.

[12] According to Barro and Lee [5] the share of skilled in the population above age 15 was .25 in the developed countries and .05 in the less developed countries in the middle 1990s. Adding to that the fact that the labor force is approximately half of this population and that most educated people work, explains the figures of .5 and .1 in the developed and less developed, respectively.

[13] See Maddison [20].

5.2. Equilibrium with partial specialization in skilled goods

This sub-section analyzes the case that the costs of production by skilled workers are equal in the two countries. In this case country B also produces some traded goods by skilled workers, but country A imports all traded intermediate goods in $[v, 1]$. Hence, the world trade equilibrium is

$$\int_{M\cap[0,v]} p(i) EX_{\mathrm{A}}(i)\, di = \int_{M\cap[v,1]} p(i) X_{\mathrm{A}}(i)\, di = P_{\mathrm{A}} Y_{\mathrm{A}} m(1-v), \tag{25}$$

where $EX_{\mathrm{A}}(i)$ is export of i from A. This trade condition, the equality of v and z_{B} and the four labor market equilibrium conditions are used in the following proposition to derive the general equilibrium in the case of partial specialization.

Proposition 3. *The equilibrium in the case of partial specialization exists and is unique. The wage ratio in the developed country* A *is the same as in the case of full specialization, namely as in Proposition* 1. *The wage ratio and technology adoption in country* B *are given by*

$$W_{\mathrm{B}} = g(z_{\mathrm{B}}) = \frac{1-h_{\mathrm{B}}}{h_{\mathrm{B}}} \frac{m + (1-m) f}{m + (1-m) f + ma \dfrac{L_{\mathrm{A}}}{L_{\mathrm{B}}} \dfrac{h_{\mathrm{A}}}{h_{\mathrm{B}}}} \frac{z_{\mathrm{B}}}{1 - z_{\mathrm{B}}}.$$

Under partial specialization the share of trade in income in country A is $m(1 - z_{\mathrm{B}})$. The share of trade in income in B is

$$\frac{z_{\mathrm{B}} m(1 - z_{\mathrm{B}})}{\dfrac{L_{\mathrm{B}} h_{\mathrm{B}}}{a L_{\mathrm{A}} h_{\mathrm{A}}}[m + f(1-m)] + (1 - z_{\mathrm{B}})m}.$$

Proof. In Appendix A. □

The equilibrium in the case of partial specialization can be described by a diagram similar to Fig. 3, with two differences. First, it does not include the $\mathbf{W_T}$ curve. Second, the $\mathbf{W_B}$ curve is different as well.

6. The effects of technical progress and trade liberalization

This section examines how skill-biased technical progress and trade liberalization affect the equilibrium in the developed and the less developed countries. Note that in the trade model we can examine not only how such changes affect wage inequality, but also how they affect trade patterns as well. We also examine, as in Section 4, how these changes affect income and productivity differences across countries. We model technical progress as increasing *f*, and trade liberalization as increasing *m*, i.e. increasing the set of traded goods. We examine the effect of these changes both in the full specialization case and in the partial specialization case.

6.1. Skill-biased technical change

Consider first the full specialization case. As is clear from Proposition 1, an increase in *f* shifts the $\mathbf{W_A}$ curve in Fig. 3 upward. Hence, it raises the wage ratio W_{A} and increases technology adoption as well. Technical progress increases the wage ratio by increasing the demand for skilled workers and by reducing the demand for unskilled. In the less developed economy B technical progress has no affect as the $\mathbf{W_B}$ curve in Fig. 3 remains unchanged and so do W_B and z_B. Hence, skill-biased technical progress has no effect on the wage ratio and on technology adoption in the less developed country.

We next examine how skill-biased-technical-change affects the patterns of trade between the developed and the less developed countries. Clearly, an increase in f shifts the $\mathbf{W_T}$ curve up and thus reduces v. As a result, country A exports less intermediate goods to B and imports more intermediate goods from it. The intuitive explanation for this result is as follows. Skill-biased technical change raises wages of skilled workers in A and thus raises the price of goods produced by them. As a result less goods are purchased by people in country B. Furthermore, since skilled workers in the developed economy earn more they purchase more and import more.

According to Proposition 1 a skill-biased technical change increases the share of trade in GDP in A, mv, and reduces the share of trade in GDP in B, $m(1-v)$. The intuitive reason is the following. Technical change does not change the wage ratio in B but it raises income of unskilled, as there is more demand for their products. Income of skilled rises as well, since imports of skilled goods in B become more expensive. Hence, both incomes rise by the same proportion. This explains why the share of trade in income declines in B. Note that income in country A rises by less, since wages of unskilled fall. Hence, while in developed countries the share of trade in GDP increases as a result of technical progress, in less developed countries this share decreases.

We next examine the effects of technical change in the case of partial specialization. First, an increase in f has the same effect on the wage ratio in country A as in full specialization and it increases the wage ratio in A. But in the case of partial specialization technical progress increases the wage ratio in the less-developed country as well. As can be seen from Proposition 3 an increase in f raises the $\mathbf{W_B}$ curve and hence raises the wage ratio and reduces technology adoption in B. The intuitive explanation is that skilled workers in A produce more goods domestically, so they export less. As a result, skilled workers in B produce more by themselves and hence their relative wage rises. Still, it is easy to see that the relative wage in A rises by much more than in B.

Consider next the effect of technical progress on international trade in this case. As the share of trade in output in A is $m(1-z_B)$, a rise in f that reduces z_B increases the share of trade in A. The share of trade in GDP in country B is described in Proposition 3. It can be shown that this share is increasing in z_B if $z_B < \frac{1}{2}$. Hence if this condition holds technical progress reduces the share of trade in income in B. Since the condition $z_B < \frac{1}{2}$ is quite likely, it is likely that a skill-biased technical change reduces the share of trade in GDP in B. We therefore conclude that the effect of skill-biased technical change on the patterns of trade is similar under full and under partial specialization.

6.2. Trade liberalization

We first examine the effects of trade liberalization in the case of full specialization. An increase in m shifts the $\mathbf{W_A}$ curve up and thus raises the wage ratio in country A. Intuitively, trade liberalization increases wage inequality in the developed country, since it increases global demand for skilled workers in this country. In the less developed country the effect is opposite. A rise in m shifts the $\mathbf{W_B}$ curve downward and thus increases z_B and lowers the wage ratio. Since trade liberalization increases global demand for unskilled workers in the less developed countries, it reduces wage inequality there. Interestingly, as wages of unskilled workers become higher relative to wages of skilled workers, more skilled technologies are adopted in the less developed country, as indicated by the increase in z_B.

We next turn to analyze the effect of trade liberalization on the patterns of trade. According to Proposition 1 the effect of trade liberalization on v is ambiguous. In the numerical example above trade liberalization reduces v, as shown in Fig. 4. In any case the change in v is small and as a result it is likely that both mv and $m(1-v)$ increase with m, as is the case in our numerical example in Fig. 4. Namely, it is likely that the shares of trade in GDP in both countries should rise as a result of trade liberalization. This differs from the effect of skill-biased technical change, which raises the share of trade in one country and reduces it in the other country.

The effects of trade liberalization under partial specialization are similar to the effects under full specialization. An increase in m raises the wage ratio in country A just as in full specialization, as shown in Proposition 3. From the same proposition it follows that a rise in m lowers the wage ratio W_B and increases z_B, as in the full specialization case. The intuition is similar: trade increases the demand for unskilled labor in the less developed countries and thus reduces wage inequality there. As for the patterns of trade, trade liberalization tends to increase the shares of trade in GDP in both developed and less developed countries. This holds always for country A, as can be deduced from Proposition 3, and in country B, if z_B is not too high.

6.3. A preliminary look at the facts

As shown above a rise in wage inequality in developed countries can be a result of either skill-biased-technical-change or trade liberalization. One way to evaluate these potential explanations is to see what happens to other variables, like the wage ratio in less developed countries or the shares of trade in GDP in the two countries.

As shown above, skill-biased-technical change raises the wage ratio in B or leaves it unchanged, but trade liberalization reduces the wage ratio in B. Hence, one way to distinguish between these explanations to the rise in wage inequality in the developed countries is to examine what happened to wage inequality in less developed countries. The evidence accumulated so far shows that in recent decades wage inequality increased in less developed countries as well, though by a smaller amount than in the developed countries. [14] It therefore indicates that the rise in wage inequality cannot be attributed mainly to trade liberalization.

Another variable we examine is the volume of trade between developed and less developed countries and its ratio to GDP in each group of countries. The model predicts that the share of this trade to GDP should rise as a result of trade liberalization in both groups of countries. But in reaction to skill-biased technical change this share is expected to rise in the developed countries only and to decline in the less developed countries. To check which explanation fits the data better, we examine the shares of trade in GDP in recent decades. This is based on a database of bilateral trade flows over the period of 1976–1999. [15] The countries have been divided into developed and less developed by using World Bank and OECD classifications. Trade flows within the two blocks were excluded from the data, so that the two groups of countries were treated as two big countries, A and B. The trade flows between the two blocks were calculated for three years: 1978, 1988 and 1998.

[14] See for example Berman et al. [7].

[15] World Bank, "Trade and Production Database, 1976–1999," available on the website: http://web.worldbank.org/WBSITE/EXTERNAL/TOPICS/TRADE/0,,contentMDK:20103741~menuPK:167374~PK:148956~piPK:216618~theSitePK:239071,00.html

It appears that the share of trade to GDP in the less developed countries has remained quite stable during these last two decades, at a level of 15–16%. The share of trade to GDP the developed countries has fluctuated. It fell during the 1980s from 3.5% to 2.7% and then increased again to a level of 3.8%. The rise is slightly stronger if South Korea is included among the developed countries instead of the less developed ones. Hence, in the recent two decades the share of inter-block trade in GDP rose slightly in the developed countries and remained stable in the less developed countries. This indicates that the effect of trade liberalization on trade between the North and the South has been rather small. It therefore further supports the above assessment that the rise in wage inequality cannot be attributed mainly to trade liberalization.

7. Income and productivity differences

In this section we return to the issue of income and TFP differentials between countries. While Section 4 focuses mainly on how these differences are affected by technical progress and human capital, this sub-section focuses on the effect of trade. To do that we begin with a world with little trade, which satisfies the condition of Proposition 2, so that full specialization prevails, and then examine how increasing m affects income and TFP differentials between the developed and the less developed countries. Note that in our framework TFP and average income are equal, so we ask whether trade increases or reduces global income differentials.

There are two ways to calculate and compare income and TFP in this model. One is to use income in terms of the same numeraire, which is a tradable good. This is equivalent to what we call in the real world income in a comparable currency. Let us call this income ratio between the two countries the dollar income ratio and denote it by $I_\$$:

$$I_\$ = \frac{P_A Y_A}{P_B Y_B}\frac{L_B}{L_A}.$$

The second way to measure income is in terms of domestic consumption, namely PPP adjusted. In our model the PPP adjusted income ratio between the two countries, which is denoted I_{PPP}, is calculated by deflating income with domestic prices:

$$I_{PPP} = \frac{Y_A}{Y_B}\frac{L_B}{L_A}.$$

Although the second measure is the one used in all recent empirical studies, the following analysis discusses both measures. We next calculate them for the case of full specialization.

Proposition 4. *In the case of full specialization, the dollar income ratio is*

$$\log I_\$ = \log\frac{v}{1-v} + \log\frac{L_B}{L_A}.$$

The PPP *adjusted income ratio is*

$$\log I_{PPP} = m\log\frac{v}{1-v} + m\log\frac{L_B}{L_A} + (1-m)\left[\begin{array}{l}\log a + \log\dfrac{1-h_A}{1-h_B} + \log\dfrac{1-z_B(1-m)}{1-(1-f)(1-m)} \\ + \int_{z_B}^{f}\log g(i)\,di - f\log W_A + z_B\log W_B\end{array}\right].$$

If v exceed f, then f replaces v in these two equations.

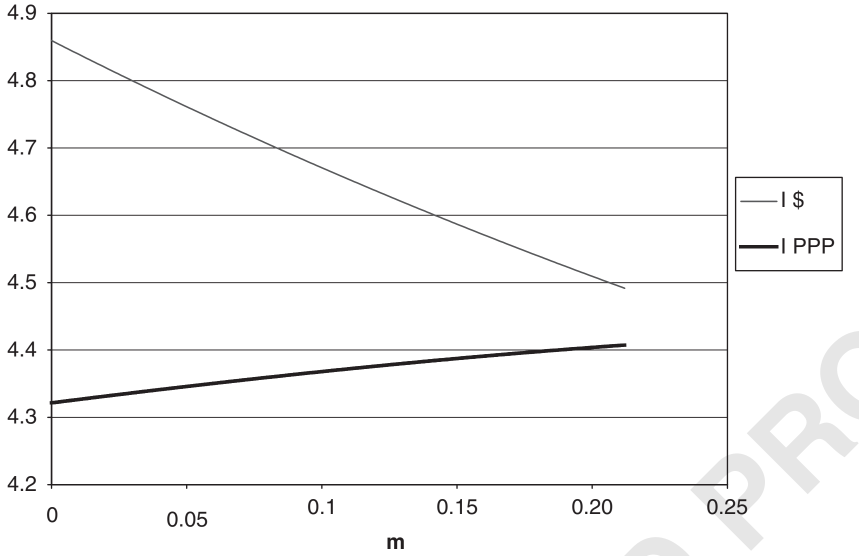

Fig. 5.

Proof. In Appendix A. □

We can next use Proposition 4 to study how technical progress and trade liberalization affect the productivity differentials. The analysis of the dollar income differential is straightforward. If v exceeds f then technical progress increases the income gap. When f exceeds v the effect switches sign and technical progress reduces the income gap as it reduces v. The effect of trade is weaker but usually reduces v as well, as shown in Section 6.1.

The analysis of the PPP adjusted income ratio is obviously much more difficult. We therefore use a numerical analysis based on the specification from Section 5.1. Fig. 5 describes the two TFP ratios between the two countries and how they are affected by trade liberalization. As the figure shows, trade increases the PPP TFP differential. Although this result does not hold for all specifications, it holds for our reasonable set of parameters. It is indeed surprising that under realistic conditions trade can increase productivity differences. The intuitive explanation is that trade leads the developed country to specialize in skilled production, which is more productive. Hence, international trade can be one of the reasons for the large productivity differences that are observed in reality. Even without trade the productivity ratio between the two countries exceeds the original productivity ratio of 3, due to the effects of human capital and technical progress. Trade further increases this productivity ratio.

8. Conclusions

This paper presents a model, which examines the interaction of skill-biased technical progress, technology adoption, human capital acquisition and international trade. It then uses the model to examine how technical progress and trade liberalization affect wage inequality in both developed and less developed countries, the patterns of trade between the two blocks of countries, and the productivity differentials between them. The model shows that skill-biased technical progress increases not only the gaps between wages of skilled and unskilled, but also the productivity gaps between countries, with and without trade. The model also shows that trade can further increase these productivity gaps, through specialization.

The model can also contribute to the debates on what caused the recent rise in wage inequality in the US and in other western economies. The data show that the patterns of trade between the rich and poor countries did not change much in the last two decades, and according to the model this means that the effect of trade liberalization has been rather small. This lends some support to the view that skill-biased-technical change has been the main force behind the rise in wage inequality.

Finally, remember that the model described in this paper is only a first shot at understanding these issues. A natural extension of this model is to add physical capital to the analysis, where machines are invented to replace workers in various tasks. This extension will better clarify the complex interactions between wages, industrialization, human capital and trade. I hope to tackle these issues in future research.

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Appendix A.

Proof of Lemma 1. The overall effect of human capital on TFP is

$$\frac{d\log TFP_{\text{LDC}}}{dh} = \frac{\partial \log TFP_{\text{LDC}}}{\partial z}\frac{\partial z}{\partial h} + \frac{\partial \log TFP_{\text{LDC}}}{\partial h}.$$

Due to (18):

$$\frac{\partial \log TFP_{\text{LDC}}}{\partial z} = \frac{\partial \log w_n}{\partial z} + \frac{\partial \log[1 + h(W-1)]}{\partial z}.$$

Due to (16):

$$\frac{\partial \log w_n}{\partial z} = -z\frac{g'(z)}{g(z)}.$$

Using (12) and (20) we get:

$$\frac{\partial \log[1 + h(W-1)]}{\partial z} = \frac{hg'(z)}{1 + h(W-1)} = \frac{(1-z)hg'(z)}{1-h} = z\frac{g'(z)}{g(z)}.$$

Hence:

$$\frac{\partial \log TFP_{\text{LDC}}}{\partial d} = 0.$$

This proves the first part of the lemma. Note that:

$$\frac{\partial \log TFP_{\mathrm{LDC}}}{\partial h}=\frac{\partial \log[1+h(W-1)]}{\partial h}=\frac{W-1}{1+h(W-1)}=\frac{z-h}{h(1-h)}.$$

This proves the second part of the lemma. □

Proof of Proposition 1. The equilibrium condition for skilled labor in A is, using Eq. (24):

$$\begin{aligned}L_{\mathrm{A}}h_{\mathrm{A}}&=\int_{[0,v]\cap M}a^{-1}s(i)[X_{\mathrm{A}}(i)+X_{\mathrm{B}}(i)]\,di+\int_{[0,f]\cap M^c}a^{-1}s(i)\;X_{\mathrm{A}}(i)\,di\\&=[vm(P_{\mathrm{A}}Y_{\mathrm{A}}+P_{\mathrm{B}}Y_{\mathrm{B}})+f(1-m)P_{\mathrm{A}}Y_{\mathrm{A}}]\frac{1}{w_{s,\mathrm{A}}}=[m+f(1-m)]\frac{P_{\mathrm{A}}Y_{\mathrm{A}}}{w_{s,\mathrm{A}}}.\end{aligned}$$

The equilibrium condition in the market for non-skilled labor in A is

$$L_{\mathrm{A}}(1-h_{\mathrm{A}})=\int_{[f,1]\cap M^c}a^{-1}n(i)X_{\mathrm{A}}(i)\,di=(1-f)(1-m)\frac{P_{\mathrm{A}}Y_{\mathrm{A}}}{w_{n,\mathrm{A}}}.$$

Similarly, the equilibrium in the market for skilled labor in B is reached at:

$$L_{\mathrm{B}}h_{\mathrm{B}}=\int_{[0,z_{\mathrm{B}}]\cap M^c}s(i)X_{\mathrm{B}}(i)\,di=z_{\mathrm{B}}(1-m)\frac{P_{\mathrm{B}}Y_{\mathrm{B}}}{w_{s,\mathrm{B}}}.$$

The equilibrium condition for non-skilled labor in B is

$$\begin{aligned}L_{\mathrm{B}}(1-h_{\mathrm{B}})&=\int_{M\cap[v,1]}n(i)[X_{\mathrm{B}}(i)+X_{\mathrm{A}}(i)]\,di+\int_{[z_{\mathrm{B}},1]\cap M^c}n(i)X_{\mathrm{B}}(i)\,di\\&=\{[m-vm+(1-m)(1-z_{\mathrm{B}})]\,P_{\mathrm{B}}Y_{\mathrm{B}}+[m-vm]P_{\mathrm{A}}Y_{\mathrm{A}}\}\,\frac{1}{w_{n,B}}\\&=[1-z_{\mathrm{B}}(1-m)]\frac{P_{\mathrm{B}}Y_{\mathrm{B}}}{w_{n,\mathrm{B}}}.\end{aligned}$$

From the labor markets equilibrium conditions we can derive the wage ratios in the two countries. The wage ratio in country A is:

$$W_{\mathrm{A}}=\frac{w_{s,\mathrm{A}}}{w_{n,\mathrm{A}}}=\frac{1-h_{\mathrm{A}}}{h_{\mathrm{A}}}\frac{m+f(1-m)}{(1-m)(1-f)}=\frac{1-h_{\mathrm{A}}}{h_{\mathrm{A}}}\frac{1-(1-m)(1-f)}{(1-m)(1-f)}.\tag{A.1}$$

The wage ratio in country B is

$$W_{\mathrm{B}}=\frac{w_{s,\mathrm{B}}}{w_{n,\mathrm{B}}}=\frac{1-h_{\mathrm{B}}}{h_{\mathrm{B}}}\frac{z_{\mathrm{B}}(1-m)}{1-z_{\mathrm{B}}(1-m)}.\tag{A.2}$$

This equation and the condition on technology adoption $W_{\mathrm{B}}=g(h_{\mathrm{B}})$ together determine the equilibrium wage ratio in country B and the degree of technology adoption as well. Note, that the RHS of Eq. (A.2) is increasing in z_{B} and hence there exists a unique intersection with g, which is decreasing in z_{B}. This intersection determines both z_{B} and the wage ratio in B.

We next describe the determination of the trade threshold v. From the labor market equilibrium conditions above and from the income ratio (24) we derive the ratio between skilled wage in A and non-skilled wage in B:

$$\frac{w_{s\mathrm{A}}}{w_{n\mathrm{B}}}=\frac{L_{\mathrm{B}}(1-h_{\mathrm{B}})}{L_{\mathrm{A}}h_{\mathrm{A}}}\frac{1-(1-f)(1-m)}{1-z_{\mathrm{B}}(1-m)}\frac{v}{1-v}.\tag{A.3}$$

This condition together with the trade condition (21) determines v. Since the right-hand side of (A.3) is increasing in v it has a unique intersection with g.

The share of trade in income in the two countries is derived immediately from (22) and (23). That concludes the proof. □

Proof of Proposition 2. If there is no trade the $\mathbf{W_B}$ curve is described by

$$\frac{1-h_B}{h_B}\frac{z_B}{1-z_B},$$

and the $\mathbf{W_T}$ curve is described by

$$\frac{L_B(1-h_B)}{aL_Ah_A}\frac{f}{1-\bar{z}_B}\frac{v}{1-v},$$

where z_B is the autarky technology adoption in B. For full specialization to prevail the $\mathbf{W_B}$ curve must be above the $\mathbf{W_T}$ curve. This holds if:

$$\frac{1-h_B}{h_B} > \frac{L_B(1-h_B)}{aL_Ah_A}\frac{f}{1-\bar{z}_B}.$$

Clearly this condition holds if:

$$1-\bar{z}_B > \frac{L_Bh_B}{L_Ah_A}\frac{f}{a}. \tag{A.4}$$

Hence if (A.4) holds full specialization prevails in the case of no trade, and due to continuity, for low values of m as well. This completes the proof. □

Proof of Proposition 3. The equilibrium condition in the market for skilled labor in country A is

$$\begin{aligned} L_Ah_A &= a^{-1}\int_{[0,v]\cap M} s(i)[X_A(i)+ex_A(i)]\,di + a^{-1}\int_{[0,f]\cap M^c} s(i)X_A(i)\,di \\ &= [vmP_AY_A + m(1-v)P_AY_A + f(1-m)P_AY_A]\frac{1}{w_{s,A}} \\ &= [m+f(1-m)]\frac{P_AY_A}{w_{s,A}}. \end{aligned} \tag{A.5}$$

The equilibrium condition in the market for non-skilled labor in country A is

$$L_A(1-h_A) = a^{-1}\int_{[f,1]\cap M^c} n(i)X_A(i)\,di = (1-f)(1-m)\frac{P_AY_A}{w_{n,A}}. \tag{A.6}$$

Hence, the wage ratio in country A under partial specialization is the same as in full specialization and is given by Eq. (A.1) as well.

The equilibrium condition for skilled workers in country B is

$$\begin{aligned} L_Bh_B &= \int_{[0,z_B]\cap M^c} s(i)X_B(i)\,di + \int_{[0,z_B]\cap M} s(i)[X_B(i)-ex_A(i)]\,di \\ &= z_B\frac{P_BY_B}{w_{s,B}} - m(1-z_B)\frac{P_AY_A}{w_{s,B}}. \end{aligned} \tag{A.7}$$

The equilibrium condition in the market for unskilled workers in country B is

$$L_B(1-h_B) = \int_{M\cap[z_B,1]} n(i)[X_B(i)+X_A(i)]\,di + \int_{[z_B,1]\cap M^c} n(i)X_B(i)\,di$$
$$= (1-z_B)\frac{P_BY_B}{w_{n,B}} + m(1-z_B)\frac{P_AY_A}{w_{n,B}}. \tag{A.8}$$

From (A.7) we can derive the value of P_BY_B and substitute it in (A.8) and get after some calculation:

$$\frac{z_B}{1-z_B}L_B(1-h_B)w_{n,B} = L_Bh_Bw_{s,B} + m(1-z_B)P_AY_A + mz_BP_AY_A$$
$$= (1-h_B)L_Bw_{s,B} + mP_AY_A.$$

From Eq. (A.5) we derive the value of P_AY_A and substitute it in the above condition. Remembering that $w_{s,B} = w_{s,A}/a$ we get:

$$\frac{z_B}{1-z_B}L_B(1-h_B)w_{n,B} = (1-h_B)L_Bw_{s,B} + \frac{amL_Ah_Aw_{s,B}}{m+f(1-m)}. \tag{A.9}$$

From (A.9) we derive W_B and get the result of Proposition 3.

We next derive the shares of trade in GDP in the two countries. The absolute volume of trade is $P_AY_Am(1-v) = P_AY_Am(1-z_B)$. Hence, the share of trade in GDP in A is $m(1-z_B)$. The share of trade in GDP in country B is:

$$\frac{P_AY_Am(1-z_B)}{P_BY_B}. \tag{A.10}$$

To calculate it use Eqs. (A.5) and (A.7) and $aw_{s,B} = w_{s,A}$ to get:

$$\frac{L_Bh_B}{aL_Ah_A}[m+f(1-m)]P_AY_A + m(1-z_B)P_AY_A = z_BP_BY_B.$$

Substituting in (A.10) we get that the share of trade in GDP in B is:

$$\frac{z_Bm(1-z_B)}{\frac{L_Bh_B}{aL_Ah_A}[m+f(1-m)] + (1-z_B)m}. \tag{A.11}$$

This completes the proof of Proposition 3. □

Proof of Proposition 4. The calculation of $I_\$$ follows from Eq. (24). To calculate I_{PPP} we devalue by the prices of the final goods. It can be easily shown that the price of the final good in country j is

$$\log P_j = \int_0^1 \log p_j(i)\,di.$$

In subtracting $\log P_{\mathrm{A}}$ from $\log P_{\mathrm{B}}$ the international prices are cancelled out and only domestic prices remain. Hence:

$$\begin{aligned}\log P_{\mathrm{B}} - \log P_{\mathrm{A}} &= \int_{M^c} \log p_B(i)\, di - \int_{M^c} \log p_A(i)\, di \\ &= \int_{[0,z_{\mathrm{B}}]\cap M^c} \log[w_{s,\mathrm{B}} s(i)]\, di + \int_{[z_{\mathrm{B}},1]\cap M^c} \log[w_{n,\mathrm{B}} n(i)]\, di \\ &\quad - \int_{[0,f]\cap M^c} \log[w_{s,\mathrm{A}} s(i)]\, di - \int_{[f,1]\cap M^c} \log[w_{n,\mathrm{A}} n(i)]\, di \\ &\quad + (1-m)\log a.\end{aligned}$$

Some further computation shows that:

$$\begin{aligned}&\log P_{\mathrm{B}} - \log P_{\mathrm{A}} \\ &\quad = (1-m)\left[\int_{z_{\mathrm{B}}}^{f} \log g(i)\, di + \log w_{n,\mathrm{B}} + z_{\mathrm{B}} \log W_{\mathrm{B}} - \log w_{n,\mathrm{A}} - f \log W_{\mathrm{A}} + \log a\right].\end{aligned}$$

We calculate $\log w_{n,\mathrm{B}} - \log w_{n,\mathrm{A}}$ by use of the labor market equilibrium conditions from the proof of Proposition 1 above and get:

$$\log w_{n,\mathrm{B}} - \log w_{n,\mathrm{A}} = \log \frac{v}{1-v} + \log \frac{L_{\mathrm{A}}}{L_{\mathrm{B}}} + \log \frac{1-h_{\mathrm{A}}}{1-h_{\mathrm{B}}} + \log \frac{1-z_{\mathrm{B}}(1-m)}{(1-f)(1-m)}.$$

Hence:

$$\log P_{\mathrm{B}} - \log P_{\mathrm{A}} = (1-m)\left[\begin{array}{l}\log a + \int_{z_{\mathrm{B}}}^{f} \log g(i)\, di - f \log W_{\mathrm{A}} + z_{\mathrm{B}} \log W_{\mathrm{B}} + \log \dfrac{v}{1-v} \\ + \log \dfrac{L_{\mathrm{A}}}{L_{\mathrm{B}}} + \log \dfrac{1-h_{\mathrm{A}}}{1-h_{\mathrm{B}}} + \log \dfrac{1-z_{\mathrm{B}}(1-m)}{(1-f)(1-m)}\end{array}\right].$$

Substituting into $I_{\$}$ ends the proof. □

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