Machines as Engines of Growth

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Abstract

This paper presents a model of growth through industrialization, where machines replace labor in a growing set of tasks. While substituting labor in some tasks, machines complement labor in the remaining tasks by increasing its productivity. This dual role of capital leads to a feedback between technical progress and wages, the cost of labor. Higher wages induce creation and adoption of machines to replace costly labor, while these machines complement labor in the remaining tasks and raise wages further. This feedback effect fuels economic growth and can even lead to long-run growth. The model shows that industrialization and growth take off only if the cost of machinery is sufficiently low. Hence, this paper suggests that the invention of the steam engine could have triggered the industrial revolution and economic growth by reducing the cost of machinery. The model also shows that growth depends positively on basic productivity and negatively on monopoly power.

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1. Introduction

This paper presents a new model of endogenous growth. It is therefore part of the effort to explain ongoing global economic growth. During the last two hundred years global output per capita has grown by more than 8 fold and in the more developed countries output per capita has grown by twice as much. This rapid growth began with the industrial revolution, somewhere around 1820, according to Maddison (1995, 2001). This paper claims that modern economic growth has been made possible by creating a new type of technologies: machines that can perform a growing set of tasks that humans have performed before, and can thus replace labor. The paper shows that the introduction of such machines can explain continuing economic growth.

This engine of growth emerges as a result of the dual effect of capital on labor in such an economy. First, capital replaces labor in more and more tasks of production. Second, capital complements labor that concentrates in the remaining tasks, by enabling labor to operate with more machines, to perform less tasks, and thus to produce more. For example, consider builders who begin to use a crane that can lift materials up the building, instead of carrying them manually. This enables the workers to finish the building more quickly, namely to increase production.

This dual effect of capital on labor, of supplementing some tasks and complementing others, creates a feedback effect between technical progress and wages, the cost of labor. On one hand, the new machines are invented and used only if they reduce the cost of production. Hence, producers compare the benefits of reducing labor to the cost of the machine. They demand the new technology only if wages are sufficiently high. On the other hand, the use of more machines complements labor in the remaining tasks and thus raises wages. This feedback between growth and wages can therefore explain how growth continues over time.

This theory can explain not only the engine of economic growth, but also what can block economic growth and what can trigger its takeoff. According to this model growth depends on three main factors. The first is the cost of machinery, which has a negative effect on industrialization. Thus, if this cost is reduced, it can trigger and start economic growth. The second factor is the overall productivity of the economy, which is assumed to be fixed overtime, and reflects geography, climate, infrastructure and access to trade. Productivity affects growth through wages. If overall productivity is low, wages might be too low to induce industrialization. A third factor is monopoly power. It is shown that if producers have monopoly power, they increase profits by paying lower wages, and that deters growth.

These results can be used to raise a few hypotheses on the potential triggers to the industrial revolution, which pushed the global economy from a rather stagnant pre-industrial equilibrium to a path of economic growth and industrialization, sometime two hundred years ago. The main possible explanation is the invention of the steam engine, which offered a new general technology to build machines. As explained above this reduction of the cost of machinery could have triggered growth. A second possible explanation could have been a rise in general productivity that could be caused by the discovery of America. Another possibility is that the collapse of Feudalism, with its established monopoly rights, and the rise of free labor markets, led to the industrial revolution by raising the cost of labor. Clearly, these possible explanations are not mutually exclusive.

The theory this paper presents on the mechanism of economic growth is strongly related to the two main existing theories of endogenous growth, that of capital accumulation, and that of technical progress. The first theory is presented by the AK models of Jones and Manuelli (1990), Rebelo (1991) and others. The second theory is the R&D based endogenous growth models of Romer (1990), Segestrom, Anant, and Dinopolous (1990), Grossman and Helpman (1991), Aghion and Howitt (1992) and Jones (1995, a, b). This paper can be viewed as an attempt to bridge these two literatures, as it has elements of both theories, but also differs from each significantly.

In this paper growth is driven by capital accumulation as in the AK models but it requires also continuous innovation. Hence, capital accumulation is combined with a continuous shift of the production function. As a result TFP growth is positive in this model, unlike the zero TFP growth in the AK models. Another important difference between this paper and usual AK models is that the share of labor in income in this model does not converge to zero in the long-run as in the AK models. These two differences therefore eliminate the problem that standard AK models do not fit well-known regularities in the historical data on TFP and on income shares.[1]

The theory this paper presents has similarity also to the R&D literature, as growth depends on continuing innovation. But this paper departs from this literature in acknowledging the fact that the cost of using a new innovation is not only the cost of invention, but also the cost of the physical machine in which the innovation is embodied.

[1] Another attempt to adjust the AK models to these empirical regularities is done by introduction of a second final good by Zuleta (2006).

As a result the effect of scale is reduced and even disappears if scale is large enough. This is an important result, since the strong theoretical scale effect of the R&D growth models is in contrast with many observed facts, as noted by Jones (1995a).

The idea of innovations that substitute labor with capital has its roots already in Hicks (1932). It then influenced the literature of induced biased innovation, as in Kennedy (1964), Samuelson (1965), and Habbakuk (1962). This literature analyzed technical change biased toward capital versus labor, but did not go far enough to examine replacement of labor by capital. This has been first modeled by Champernowne (1963), who studied its effect on the aggregate production function. This idea appears again in Zeira (1998), which models technology adoption as using machines that replace workers, in order to study output differences across countries.[2] The current paper uses this idea in a very different framework, of global growth, and also adds to the analysis endogenous invention of technologies.[3]

The paper is constructed as follows. Section 2 presents the basic idea through a simple model. Section 3 describes the main results of the model. Section 4 discusses possible triggers to the industrial revolution and to the beginning of modern economic growth. Section 5 compares the model with the two main theories of endogenous growth. Section 6 examines optimal growth. Section 7 discusses two extensions of the theory, one on divergence and one on the role of energy in growth. Section 8 summarizes and an Appendix contains mathematical proofs.

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[2] Similar models are used by Zeira (2006) and Caselli and Coleman (2006) to study replacement of unskilled by skilled workers. Beaudry and Collard (2002) use a similar idea as well, in analyzing employment dynamics. A related approach is that of 'appropriate technologies' of Basu and Weil (1998).
[3] Recently, a number of new papers, which have been written independently, apply this approach to the issue of endogenous growth. These are Zuleta (2005), Peretto and Seater (2006), and Givon (2006). This paper differs from these in its focus on the role of wages and of the cost of machinery and in its micro-foundations.

2. A Simple Model of Technical Progress

We present the main idea of the paper by use of a very simple model, which is later extended in a few directions. Consider an economy with one final physical good, which is used both for consumption and for investment. Before it reaches its final use, some of it is lost, through transportation, commerce and similar effects, so that only a share a reaches final use, where $a < 1$. The parameter a therefore measures the basic productivity of the economy, which reflects geography, climate, quality of land, infrastructure, access to trade and similar factors. The good is produced by labor and capital, where capital consists of structures, tools, and also machines, which replace some of the tasks earlier performed by labor. For simplicity assume a production function of fixed proportions.[4] Production of 1 unit requires l units of labor and k units of capital. Time is discrete and a time period is assumed to be long enough for capital to fully depreciate within a period.

Technical progress enables producers to replace labor by machines, namely to reduce l at the expense of increasing k. This negative relationship between k and l is described by a decreasing function θ:

$$k = \theta(l), \quad \text{for all } l \leq 1.$$

The domain of the function θ implies that the initial pre-industrial technology requires 1 unit of labor and $\theta(1)$ units of capital, which consist of structures, some tools and no machines. The function θ is described in Figure 1 below and it satisfies: $\theta'(l) < 0$. The pre-industrial technology is described by the point PI in the figure. It is further assumed that the innovation of new machines in each period is costless, so that the only cost of

[4] This assumption is not critical for the main results of the paper.

adopting new technologies is the cost of purchasing the machines, namely purchasing capital.[5]

We next analyze the demand for new technologies. Let the final good be the numeraire. The real wage is denoted by w. For simplicity assume that the interest rate is constant over time, which holds if consumers are risk neutral.[6] Let R denote the gross interest rate, $R = 1 + r$, which is also equal to the sum of the interest rate and the depreciation rate. The cost of production of 1 unit of the good for final use is:

$$(1) \qquad \frac{1}{a}\left[wl + R\theta(l)\right].$$

A necessary condition for technical progress to take place is that the introduction of new machines reduces the cost of production, namely that the cost function (1) is increasing with l. Hence, technical progress continues as long as:

$$(2) \qquad \frac{w}{R} > -\theta'(l).$$

This condition implies that higher wages induce technical progress. This is one part of the feedback between wages and technical progress, which is described in the introduction.

We turn next to describe how wages are determined. Assuming perfect competition and free entry, the cost of production is equal to the price of the final good, which is the numeraire, and thus equal to 1. Hence:

$$\frac{1}{a}\left[wl + R\theta(l)\right] = 1,$$

or:

$$(3) \qquad \frac{w}{R}l + \theta(l) = \frac{a}{R}.$$

[5] Sub-Section 5.2 introduces costly innovation to the analysis.

[6] The case of risk aversion is analyzed in an extension of the model in Section 6.

Equation (3) describes how the real wage is determined and actually how income is divided between the two factors of production. Graphically it is presented in Figure 1. If the economy is in point X then the lower part of the vertical line that passes through X is $\theta(l)$ and the upper part is wl/R.

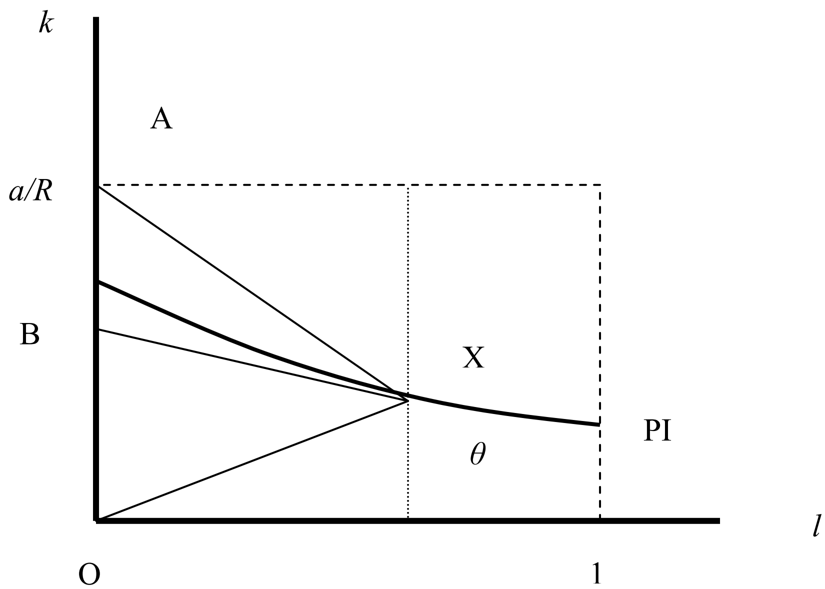

Figure 1

Hence, the ratio of wages to gross interest rate, w/R, is equal to the slope (in absolute terms) of the line that connects point A with point X. Hence, this figure also shows whether condition (2) to technical progress holds. As long as the line AX crosses the θ curve from above, as in Figure 1, condition (2) is satisfied and technical progress, namely reduction of l, continues. Note that if AX crosses θ from above and l is reduced, the slope of the line AX increases as well, namely technical progress increases the wage rate w and the ratio w/R. This is therefore the second part of the feedback effect: technical progress raises wages and thus increases further the incentive for technical progress.

So far we have described the demand for new technologies. We next turn to describe the supply of technical progress. As the economy moves along the θ curve to the left, the rate of reduction of labor requirement l depends on the ability to develop new machines, or new technologies. It is assumed that this is a gradual process, namely that "windows" cannot be invented before a computer is invented, or that the airbag cannot be invented before first inventing the car, etc. One way to formalize this is to assume that in each period the labor requirement l can be reduced only by some proportion δ. Hence, as long as technical progress continues:

$$l_t = l_{t-1} - \delta l_{t-1} = l_{t-1}(1-\delta). \tag{4}$$

Equation (4) implicitly assumes that as long as there is demand, technical progress continues at a fixed rate.[7]

3. Endogenous Economic Growth

This section examines the growth dynamics of the economy. Assume that the labor force in the economy is L. Thus, output, output per worker, capital, and the capital labor ratio in this economy are described by:

$$Y = \frac{aL}{l},\ y = \frac{Y}{L} = \frac{a}{l},\ K = \frac{Y}{a}\theta(l),\ \frac{K}{L} = \frac{\theta(l)}{l}. \tag{5}$$

Clearly, as the labor input l is reduced with technical progress, output and output per worker increase. Thus, technical progress leads to economic growth. Note that the capital

[7] One way of modeling the possibility that technical progress might come to a stop at some point is that invented machines become too expensive. Namely it is reflected in the function θ.

labor ratio is equal to the slope of the line that connects the origin O to point X. Thus it also increases as l is reduced and the economy grows. Equation (5) also highlights the double effect of capital on labor in this model. On the one hand capital replaces labor in a growing number of tasks. On the other hand, it is complementary to the workers who concentrate in the remaining tasks, as their productivity increases. Hence capital both substitutes labor and complements it.

From equation (5) it also follows that the rate of growth of the economy is determined by the rate of reduction of the labor requirement l. From (5) and (4) we get that the rate of growth of output and of output per worker in period t is equal to:

$$(6) \qquad g_t = \frac{\frac{a}{l_t}}{\frac{a}{l_{t-1}}} - 1 = \frac{l_{t-1}}{l_t} - 1 = \begin{cases} \frac{\delta}{1-\delta} & \text{if there is technical progress,} \\ 0 & \text{if not.} \end{cases}$$

If we denote $\delta/(1-\delta)$ by g, then the rate of growth is equal to g if there is technical progress and to 0 otherwise.[8] Hence, as long as growth continues, the rate of growth is fixed and it depends only on the technological ability to invent new machines.

Next consider the conditions under which economic growth takes place. As shown in Section 2 economic growth and technical progress take place if the line AX crosses the θ curve from above. This happens either if a is sufficiently high, or if θ is sufficiently low.[9] Hence, growth materializes either if the cost of machinery is low or if productivity is high. Similarly the model can help in better understanding the possibility that growth will not continue forever. If at some future point in time new machinery will

[8] More precisely, if technical progress comes to a stop, growth might be smaller than g in the last period of growth, if θ is convex.
[9] Clearly a low interest rate is also a possible condition of growth, but we assume that it is fixed, which is a reasonable assumption in such a long-run model.

become increasingly more expensive, as shown in Figure 2, economic growth might come to a stop. In that case the economy will get stuck at a point like B, where $w/R = \theta'(l_B)$, and technical progress and growth will stop.

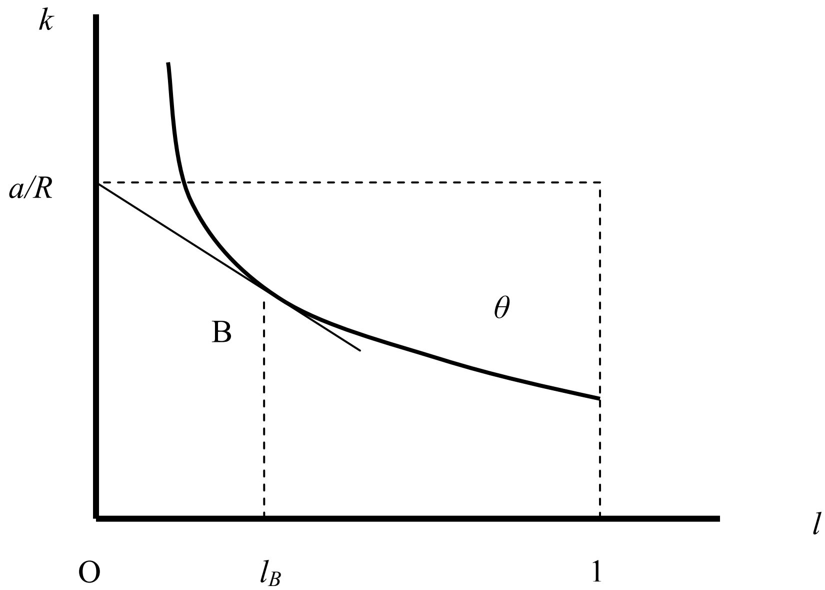

Figure 2

Before turning to discussion and extensions of the basic model, consider a simple but illuminating case, where the function θ is linear:

(7) $$k = \theta(l) = s + m(1-l) = s + m - ml.$$

The interpretation of this function is that s is capital requirement in the pre-industrial technology, which consists mainly of structures, while m is the amount of machinery that replaces each unit of labor. In this case the condition for economic growth, namely that the AX line crosses θ from above, becomes:

(8) $$a > R(m+s).$$

If this condition holds, growth continues, while if it does not hold, growth does not take off and the economy remains unindustrialized. Note, that (8) depends positively on productivity *a* and negatively on the cost of machinery *m*.

4. What Can Trigger Economic Growth?

As Maddison (1995) shows, economic growth is a fairly recent historical phenomenon that started in the beginning of the 19th century and has been going steadily since then. Economic growth is also inherently related to the process of industrialization. What could have triggered this process that changed society and economy so dramatically in the last two centuries? We show in this section that our model can offer a number of potential answers to this question.

We begin by considering an economy that is in an initial pre-industrial equilibrium and then we review three potential types of exogenous changes that can hit the economy and shift it into an equilibrium path of economic growth. The first one is a reduction of the cost of machines θ. Figure 3 below shows how such a change, from θ_0 to θ_1, pushes the economy from the pre-industrial equilibrium PI into a process of industrialization and growth along the new curve θ_1.

What historical event could have lowered the cost of machinery so drastically? The most suitable candidate is the invention of the steam engine by the end of the eighteenth century. This was not only an invention of one machine, but what we call a "general purpose technology." It signaled to all that there is a new way to produce goods, which is by machines instead of labor, by thermal energy instead of human energy. It also signaled that this new way is not too costly, but is realistic and available. By shifting the cost of machinery from θ_0 to θ_1, this invention pushed the economy into a path of industrialization and growth.

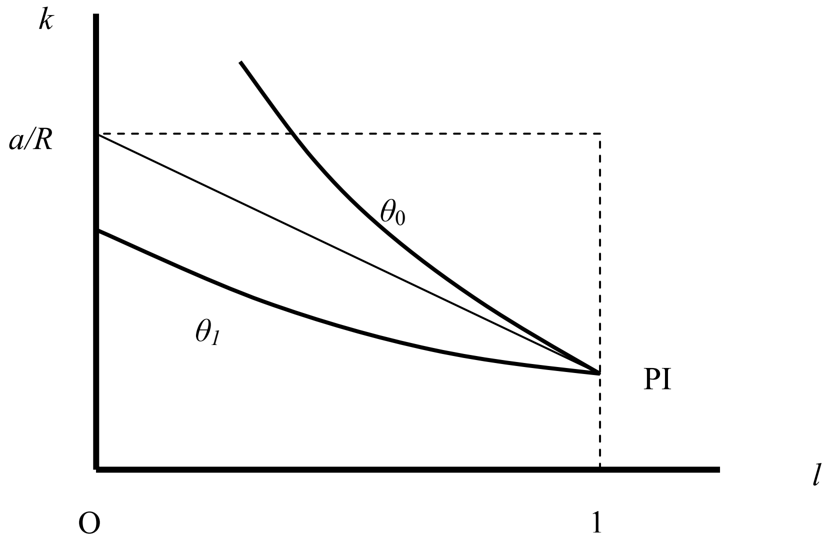

Figure 3

A second exogenous change that can shift the economy from a pre-industrial equilibrium into a growth path is a rise in the overall productivity *a*, as shown in Figure 4 below. Productivity increases from a_0 to a_1 and the economy moves from the pre-industrial equilibrium to a growth path along θ. Such a rise in productivity increases the cost of labor and thus creates incentives to invent machines that replace labor. What can be the historical equivalent of an increase in productivity prior to the industrial revolution? One possible event is the discovery of America, which contributed to sea faring, to agriculture, through new plants and animals, and also added new territories. It raised incomes and as a result the cost of labor increased as well. The rise in income after the discovery of America is documented in Maddison (2001). Between 1500 and 1820 income per capita in Western Europe increased by more than 60%. Hence, the discovery of America, and the rise in productivity it caused, could be one potential trigger to the beginning of the industrial revolution.

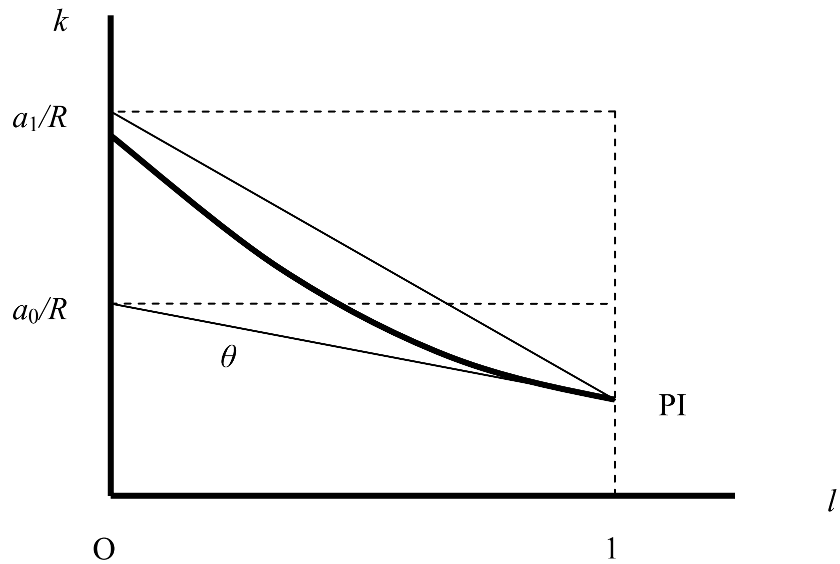

Figure 4

A third possible explanation to what could have triggered economic growth is the collapse of feudalism, which reduced monopoly power in the economy. This collapse started in England with the Cromwell Revolution, was accelerated by the spread of Protestantism, and was further intensified with the French Revolution and the Napoleonic wars. During the 19th century all over Europe the old system of control by few over land and production continued to crumble down. Our model claims that such a historical development could also trigger the industrial revolution. To show it we incorporate next monopoly power in the model.

Assume that producers have monopoly power enabling them to earn a profit, which equals a share *z* of revenues. Such profits arise for example when there is a finite number of producers, N, who participate in Cournot competition. If $N > 1$, the profit of each producer in a symmetric equilibrium is equal to a share z of revenues, where: $z = 1/N$. If producers have a profit of size z of revenues, the price is not equal to the cost of production, but is higher and hence the equilibrium condition becomes:

$$1 = \frac{wl + Rk}{a(1-z)}.$$

Hence, instead of equation (3) the equilibrium in this economy is described by:

$$\text{(9)} \qquad \frac{w}{R} l + \theta(l) = \frac{a(1-z)}{R}.$$

The equilibrium diagram is the same as in Figure 1, except that a/R is replaced by $a(1-z)/R$. The effect of a decline in monopoly power can therefore be described by diagram similar to Figure 4, except that instead of an increase in a, the horizontal line shifts up as a result of a decline in monopoly power z. Hence, the collapse of feudalism could also have contributed to the onset of the industrial revolution by raising labor costs.

The scope of this paper is of course not wide enough to include an empirical examination of the role of each of these three events as triggers to the industrial revolution. It only points at the theoretical possibility of three types of triggers: a reduction in the costs of machinery, a rise in overall productivity and a collapse of monopoly power. Hence, the invention of the steam engine, the discovery of America and the collapse of feudalism, are all potential suspects according to this theory of growth.

5. Comparison with the Endogenous Growth Literature

In this paper economic growth is driven by both capital accumulation and by continuous invention of machines. Hence it shares some elements with both the AK models of endogenous growth and with the R&D based endogenous growth models. But this paper also differs significantly from these two lines of research. This section discusses both the similarities and the differences.

5.1 Comparison with AK Models

This model enables us to derive a long-run production function where output depends on capital, which has similarity to the production function in AK models. It can be shown that by making l a function of capital K, equation (5) yields the following long-run relationship:

$$K = \frac{Y}{a}\theta\left(\frac{aL}{Y}\right). \tag{10}$$

In the case of the linear θ function of equation (6), the long-run production function is:

$$Y = \frac{a}{s+m}K + \frac{ma}{s+m}L. \tag{11}$$

This long-run production function is similar to the AK production function of Jones and Manuelli (1990) and Rebelo (1991), but it differs in an important aspect. This long-run production function is an envelope of many temporal production functions, where technical progress shifts the economy from one production function to the other as more machines are invented. As a result the marginal productivity of capital in each period is not equal to the long-run marginal productivity of capital, as in the AK models. To show this, note that equation (10) implies that the long-run marginal productivity of capital is equal to:

$$LRMPK = \frac{a}{\theta(l) - l\theta'(l)}.$$

The denominator $\theta(l) - l\theta'(l)$ is equal exactly to the length of the segment OB in Figure 1. Hence, this figure implies that if the there is technical progress $LRMPK > R$. This can be seen from the linear case as well, since $MPK = a/(s+m)$ and if there is technical progress and condition (8) holds, the long-run marginal productivity of capital exceeds the short-run.

This result leads to important deviations from the AK models. First according to the AK models the share of capital in income is equal to one or converges to one. This result is bothersome, since a host of accumulated empirical data points at a share of capital that is much lower than 1, actually around 1/3. In our model the share of capital in income is equal to:

$$s_K = \frac{RK}{Y} = \frac{\theta(l)}{a/R}.$$

According to Figure 1, as long as there is technical progress and growth, this share is smaller than 1 and it converges to a share which is smaller than 1.

This difference is important also because of the many observations, since Solow (1957), on the strong growth of total factor productivity TFP. If the share of capital in income is 1 or close to 1, the rate of growth of TFP is zero or close to zero. This is another problem in fitting AK models to the data. Again, in this paper TFP growth is positive. This raises the question what makes this model differ so much from the AK models. There are two sources to this difference. One is that in this model the long-run production function is an envelope of many temporal production functions, as explained above. The second is the bound on technical progress in each period, namely that not too many machines can be invented each period. This bound affects the marginal productivity of capital, since in this model the choice of technology and the choice of capital input are strongly related.[10]

5.2. Costly Innovation: Comparison with R&D Based Growth Models

This sub-section compares this model with the R&D based endogenous growth models. Similar to these models, growth in this paper is also driven by new innovations. But the existing R&D models focus only on the innovation cost of new technologies, while this paper emphasizes the capital cost of the machines within which the new technologies are embodied. So far the model has assumed for simplicity that the cost of innovation is zero. It next examines what happens if the cost of innovation is positive, so the comparison with R&D based growth models becomes more transparent.

Consider the model presented in Section 2 with one extension that inventing a new machine is costly. The cost of innovation depends both on the size of the invented machine and on the alternative cost of resources, namely on output per capita.[11] Hence, the cost of innovation in period $t-1$, which is when the machines used in t is invented, is:

$$\Delta k_t \frac{bY_{t-1}}{L}. \tag{12}$$

For simplicity assume that a patent lasts only one period and in next periods it becomes public knowledge. Hence, a machine, which is invented in period t-1, costs Δk_t in future periods, but $\Delta k_t + z_{t-1}$ in the first period, namely in t–1, where z is the patent fee. Due to competition between innovators the patent fee for an invented machine is equal to the cost of innovation divided by the amount of machines, namely by output:

[10] Zuleta and Young (2006) finds another way to derive share of capital smaller than 1 in AK models, by adding another final good, which has no technical progress.
[11] This is of course a simplifying assumption. Alternative assumptions on cost, like wages, yield similar results.

$$z_{t-1} = \frac{\Delta k_t b Y_{t-1}}{L Y_t} = \frac{\Delta k_t b}{L(1+g)}.$$

Technical progress occurs if it reduces the costs of production, namely if:

$$\Delta l_t w_t > R(\Delta k_t + z_{t-1}) = R\Delta k_t\left(1 + \frac{b}{L(1+g)}\right).$$

From this equation it is derived that the condition for technical progress is:

(13) $$\theta'(l) < \frac{w}{R}\frac{L(1+g)}{L(1+g)+b}.$$

This condition implies that with costly innovations there is a scale effect, and a larger scale L can speed innovations and growth. But the scale effect in this model is diminishing, and as L becomes large the scale effect becomes negligible and the model converges to the benchmark model from Section 2. Thus, the scale effect in the original R&D based growth models, which does not fit the data well, as shown by Jones (1995a), is much reduced here. The intuitive reason is straightforward. The cost of adopting an innovation is the sum of the cost of innovation and the cost of the physical machine, in which the innovation is embodied. As scale increases, the cost of innovation per user falls, but the capital cost of the machine remains unchanged. Hence, the benefit from scale is diminishing. Thus, scale can help economic growth, but only to a limited and diminishing extent.

6. Endogenous Interest Rates and Optimal Growth

In this section the basic production model from Section 2 is embedded within an optimizing Ramsey model. This extension provides both a more realistic case of variable interest rates and also examines the optimality of the equilibrium growth path. Consider a discrete time economy with a mass of size L of infinite horizon individuals. Each of them supplies 1 unit of labor in each period of time and derives utility from consumption of the final good:

(14) $$U_0 = \sum_{t=0}^{\infty} \frac{\log c_t}{(1+\rho)^t}.$$

Hence, with respect to consumption and saving this is a standard Ramsey economy. The production side of the economy is the same as in the basic model in Section 2, with a linear function θ, as in equation (7). Finally, assume that there is perfect competition in all markets and that agents form their expectations rationally.

Denote the wage rate at time t by w_t and the gross interest rate paid in period t by R_t. Hence, due to condition (2) machines are invented and adopted for period t if:

(15) $$\frac{w_t}{R_t} > m.$$

From (3) and (7) we get that:

$$\frac{w_t}{R_t} = \frac{\frac{a}{R_t} - s - m + ml_t}{l_t} = \frac{\frac{a}{R_t} - s - m}{l_t} + m.$$

Hence, technical progress takes place in period t if:

(16) $$R_t < \frac{a}{s+m}.$$

The dynamic solution of the model must also satisfy the two well-known conditions of a representative agent economy. One is the Euler condition, namely the first order condition of utility maximization:

(17) $$\frac{c_{t+1}}{c_t} = \frac{R_{t+1}}{1+\rho}.$$

The second is the goods market equilibrium condition:

(18) $$c_t = \frac{Y_t - K_{t+1}}{L}.$$

We next explore the dynamic Rational Expectations solution to these two dynamic equations with the condition of technical progress, which satisfies the No-Ponzi-Game condition.

The amount of output is deduced from equation (5): $Y_t = aL/l_t$. Investment is also calculated by use of (5): $K_{t+1} = L(s + m - ml_{t+1})/l_{t+1}$. Hence, applying (6) and (18) we get that the consumption per worker is equal to:

$$c_t = \frac{a + ml_t - (s+m)(1+g_{t+1})}{l_t}.$$

Substituting in the Euler condition (17) we calculate the interest rate:

(19) $$R_t = (1+\rho)(1+g_t)\frac{a + ml_t - (s+m)(1+g_{t+1})}{a + ml_{t-1} - (s+m)(1+g_t)}.$$

Substituting in (16) we get that the condition for technical progress in period *t* is:

(20) $$(1+\rho)(1+g)\frac{a + ml_{t-1}/(1+g) - (s+m)(1+g_{t+1})}{a + ml_{t-1} - (s+m)(1+g)} < \frac{a}{s+m}.$$

Equation (20) leads to the following proposition.

Proposition 1:

The equilibrium growth path satisfies:

a. A necessary condition for sustained growth is $a \geq (1+\rho)(1+g)(s+m)$.

b. If $a < (1+\rho)(1+g)(s+m)$, economic growth never takes off.

c. If $a \geq (1+\rho)(1+g)(s+m)$, economic growth can go forever, but it can also stop at some future period *T*.

d. If: $a\dfrac{a-(s+m)(1+g)}{a-(s+m)} \geq (1+\rho)(1+g)(s+m)$, economic growth never stops.

Proof: in the Appendix.

If an economy is on a path of sustainable economic growth and $g_t = g$ in every period *t*, then unit labor requirement is falling continuously and converging to 0. Output rises at rate *g* or and the gross interest rate is equal to:

$$R_t = (1+\rho)(1+g)\frac{a + ml_{t-1}/(1+g) - (s+m)(1+g)}{a + ml_{t-1} - (s+m)(1+g)}.$$

It can be shown that as the economy grows and l_{t-1} falls to zero, the interest rate rises and converges to $(1+\rho)(1+g)$.

We next examine the optimality of economic growth. The intertemporal utility of the representative agent in this economy is equal to:

(21) $$\sum_{t=0}^{\infty}(1+\rho)^{-t}\log\left[\frac{a}{l_t} - \frac{s+m}{l_{t+1}} + m\right].$$

<u>Proposition 2:</u> The market equilibrium growth path is optimal.

Proof: in the Appendix.

Note that according to Proposition 2 it might be optimal to remain undeveloped and to avoid industrialization if basic productivity *a* is too low, or if the cost of machinery *m* is too high. This reflects the fact that economic growth is costly, as it requires investment in machines. The question arises whether it is beneficial to invest in infrastructure in order to increase productivity *a* and move from stagnation to growth. The answer to this question is not straightforward, since welfare, namely the optimal value of (21), is a continuous function of productivity and there is no big jump in welfare when growth begins.

7. Extensions

This section discusses briefly two extensions to the model that have interesting implications. The first one shows that the model can account not only for global growth, but also for the great divergence between regions since the industrial revolution. The second shows that energy prices can have a negative effect on the possibility of long-run economic growth. Note that energy is strongly related to this model, since replacing workers by machines also replaces human energy by thermal energy.

7.1. Divergence between Regions

So far this model has been used to describe global economic growth, namely it implicitly assumes that the closed economy is the world. Next we show that the model can be applied to explain large and growing differences across countries. As shown in Maddison (1995), Pritchett (1998), Bourguignon and Morrison (2002) and many other studies, gaps between regions have been increasing significantly since the beginning of the industrial revolution. This section shows how this model can account for such findings.[12]

[12] This sub-section extends the Zeira (1998) results by adding endogenous innovation.

Consider a world with two countries, or regions, *A* and *B*. The two countries are similar except in their basic productivity *a*, and it is assumed that $a_A > a_B$. Furthermore, assume that the θ function is linear and that the productivities in the two countries satisfy:

$$a_A > R(m+s) > a_B. \quad (22)$$

Assume that there is full capital mobility in the world, so the gross interest rate *R* is the same for both countries. The equilibrium in this economy is straightforward. Due to condition (22) economy *A* grows at a fixed positive rate *g*. Economy *B* is stagnant and is stuck at the pre-industrial level.

This model can therefore account for very different regional growth performance, due to disparities in basic productivity. It is interesting to examine the data presented by Maddison (2001) with respect to two main regions. Region A is Western Europe, Western Offshoots and Japan. Region B is the rest of the world. In 1500 GDP per capita in A was 704 (measured in 1990 dollars), while GDP per capita in B was 535. Until 1820 GDP per capita in A rose to 1,130, a rise of 60%, while GDP per capita in B rose only to 573, a rise of 7%. This shows that at the outset of the industrial revolution the productivity difference between the regions was already significant. This difference can explain the later divergence.

Similarly, the model can be applied to analyze differences in costs of machines, in addition to differences in productivity. A country that faces a high cost of machines, due to import costs, has a higher *m* and as a result growth slows down and may even stop completely. This result of the model is related to the empirical finding of Barro (1991) and other recent cross-country studies, who find that high costs of investment goods have a strong negative effect on growth.

7.2. Energy and Growth

It is important to remember that machines that replace humans in various jobs require energy to operate. Hence machines that replace workers also replace the source of energy, from human energy to fossil energy, being either coal or oil. Thus, if we want to model the process of replacing workers by machines more realistically, we should add the energy requirements. Next we extend the model in this direction in a very simple way. Assume that for each unit labor requirement l the capital requirement is $k = \theta(l)$ and the energy requirement is $e = \varepsilon(l)$, where both θ and ε are decreasing functions. Denote the price of energy by q, and assume that it is fixed over time. The condition for industrialization and economic growth is similar to the basic model:

(23) $$\frac{w}{R} > -\theta'(l) - \frac{q}{R}\varepsilon'(l).$$

Similarly the condition that determines the wage level is:

(24) $$\theta(l) + \frac{q}{R}\varepsilon(l) + \frac{w}{R}l = \frac{a}{R}.$$

It follows that the equilibrium is described by a diagram similar to the one in Figure 1, except that the curve $\theta(l)$ is replaced by $\theta(l) + q\varepsilon(l)/R$. Hence the condition for economic growth is that the line from A crosses this curve from above. The condition for growth therefore depends on a low price of energy q. If this price rises by too much growth can come to a stop. Hence, this model implies that the growth process is inherently bounded by the supply of energy on our planet. Of course, we can assume that the stock of energy on our planet is large enough, and that even when it is depleted we will be able to find other ways of harnessing solar energy for our use. But this brief analysis demonstrates that the price of energy is crucial for the process of industrialization and economic growth.

8. Summary

This paper presents a model of industrialization, and describes it as a process of inventing new machines that replace labor in a growing set of tasks. In this process the wage rate plays a critical role. Wages serve as an incentive for adopting new technologies. But wages are also positively affected by these technologies, since workers who operate with more machines become more productive. This feedback between wages and technology is the main mechanism that drives the results of this paper. It explains how the growth process can continue for long periods, it explains how growth is so sensitive to the cost of machinery, to productivity, and it also explains why monopoly deters growth.

Finally, it is time to briefly discuss the type of innovations in this paper, namely machines that replace human labor. Although this is only one specific type of innovation, it can be shown to be quite common and general. Even an innovation that replaces a machine by a better machine also enables the workers operating it to use less labor in production. Furthermore, even innovations of new consumption goods tend to replace labor in one way or another. A dishwasher, TV dinner, radio, cinema, all replace labor, either at home, or in the workplace. We do not have to go back in history to the Ludites, to realize that new machines that replace human labor have had a central role in economic growth since the industrial revolution. This paper shows that embodying this insight into growth theory can help us significantly in understanding the growth process.

Appendix

Proof of Proposition 1:

Consider first an economy that grows forever, namely it has technical progress in every period t. In that case condition (20) implies that for all t:

$$\text{(A.1)} \qquad (1+\rho)(1+g)\frac{a+ml_{t-1}/(1+g)-(s+m)(1+g)}{a+ml_{t-1}-(s+m)(1+g)}<\frac{a}{s+m}.$$

Since $l_{t-1}\xrightarrow[t\to\infty]{}0$, the left hand side of (A.1) converges to $(1+\rho)(1+g)$. Hence, $a\geq(s+m)(1+\rho)(1+g)$ and this is a necessary condition for sustained growth. This proves part a. of the proposition.

As a result, if $a<(s+m)(1+\rho)(1+g)$ growth does not continue for ever. Let T be a period in which the economy does not grow. If the economy grows in T-1 we get:

$$(1+\rho)(1+g)\frac{a+ml_{T-2}/(1+g)-(s+m)}{a+ml_{T-2}-(s+m)(1+g)}<\frac{a}{s+m}.$$

But note that:

$$(1+\rho)(1+g)\frac{a+ml_{T-2}/(1+g)-(s+m)}{a+ml_{T-2}-(s+m)(1+g)}=$$
$$=(1+\rho)\frac{a(1+g)+ml_{T-2}-(s+m)(1+g)}{a+ml_{T-2}-(s+m)(1+g)}>(1+\rho)(1+g)>\frac{a}{s+m}.$$

This is a contradiction. Hence, if $a<(s+m)(1+\rho)(1+g)$, the economy does not grow in any period and growth does not take off. This proves part b. of the proposition.

Next assume that:

$$\text{(A.2)} \qquad a\frac{a-(s+m)(1+g)}{a-(s+m)}\geq(1+\rho)(1+g)(s+m).$$

As a result condition (20) implies that even if there is no growth in period t+1, then in period t the economy grows, since:

$$
\begin{aligned}
&(1+\rho)(1+g)\frac{a+ml_{t-1}/(1+g)-(s+m)}{a+ml_{t-1}-(s+m)(1+g)}= \\
&=(1+\rho)\frac{a(1+g)+ml_{t-1}-(s+m)(1+g)}{a+ml_{t-1}-(s+m)(1+g)}< \\
&<(1+\rho)\frac{a(1+g)-(s+m)(1+g)}{a-(s+m)(1+g)}\leq\frac{a}{s+m}.
\end{aligned}
$$

Clearly, if there is growth in period t+1, there is growth in t. Hence, under condition (A.2) economic growth never stops. This proves part d. of the proposition.

Finally, consider the intermediate case, where:

$$
\text{(A.3)} \qquad a\geq(1+\rho)(1+g)(s+m)>a\frac{a-(s+m)(1+g)}{a-(s+m)}.
$$

In this case a path of continuous growth (in each period) is an equilibrium path, since:

$$
\text{(A.4)} \quad (1+\rho)(1+g)\frac{a+ml_{t-1}/(1+g)-(s+m)(1+g)}{a+ml_{t-1}-(s+m)(1+g)}<(1+\rho)(1+g)\leq\frac{a}{s+m}.
$$

Furthermore, (A.4) implies that if there is growth in period T, there is growth in all previous periods as well. We next show that under condition (A.3) growth can stop at some future period. The condition for growth in T-1, while there is no growth in T, is::

$$
(1+\rho)(1+g)(s+m)\frac{a+ml_{T-2}/(1+g)-(s+m)}{a+ml_{T-2}-(s+m)(1+g)}<a.
$$

Technical progress reduces l and thus the LHS of this equation increases. At some period T* it must surpass a, as implied by (A.3). Hence, growth until period T* and not growth from T* on is an equilibrium. This proves part c. of the proposition.

Proof of Proposition 2:

A central planner should maximizes (21) by choosing for each period of time either $l_{t+1} = l_t$ or $l_{t+1} = l_t/(1+g)$. Technical progress occurs in period t+1 if the derivative of (21) with respect to l_{t+1} is negative. This derivative is equal to:

$$\frac{\frac{s+m}{l_{t+1}^2}}{\frac{a}{l_t} - \frac{s+m}{l_{t+1}} + m}(1+\rho)^{-t} - \frac{\frac{a}{l_{t+1}^2}}{\frac{a}{l_{t+1}} - \frac{s+m}{l_{t+2}} + m}(1+\rho)^{-t-1}.$$

Hence, the derivative is negative if and only if:

$$\frac{s+m}{c_t}(1+\rho) - \frac{a}{c_{t+1}} < 0.$$

This is equivalent to:

$$(1+\rho)\frac{c_{t+1}}{c_t} < \frac{a}{s+m}.$$

Due to (15) and (16) this is the same condition as in the market equilibrium. Q.E.D.

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