Review of Economic Studies (1994) **61**, 31-44 0034-6527/94/00030031$02.00

Informational Cycles

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First version received April 1992; *final version accepted September* 1993 (*Eds.*)

This paper shows that if demand is unknown and continuously changing and if investment is costly, then output and investment are cyclical. The cycles are generated by changes in information over time, as investors increase production and thus accumulate more information about demand. These are, therefore, informational cycles. The paper also shows that the frequency of cycles depends positively on profitability and negatively on the rate of interest.

1. INTRODUCTION

This paper shows that the process of search for investment opportunities in a continuously changing economy can generate cycles in output and investment, due to informational dynamics. This result holds both for individual sectors and for aggregate economic activity.

The main result of the paper is presented by a simple model of a market for one good, where demand changes randomly over time. The maximum quantity demanded, which we shall call "potential demand", is assumed to follow a random walk. This potential demand is unknown until it is reached. Hence, the demand curve can be revealed only by producing more, namely by investing. Since investment is costly, output expands gradually, as firms try to reduce the expected costs of over-investment. But as time passes, firms enter the market more rapidly, since changes in demand add new profit opportunities. Soon, entrepreneurs hit maximum demand and after that investment slows down again. Since demand keeps changing, this happens over and over again and output and investment go through repeated cycles. These cycles are generated by changes in investors' information on demand, and hence we call them "informational cycles". It is shown in the paper that cycles are similar, except for length, which is random. It is further shown that cycle frequency depends positively on profitability and negatively on the rate of interest.

As the paper shows, three basic assumptions account for informational cycles. The first is that investment is costly, the second is that demand is unknown, and the third is that demand changes continuously. This paper therefore continues the line of research of Zeira (1987), Rob (1991) and Caplin and Leahy (1993) on investment in markets of unknown size.[1] These papers show that under the first two assumptions, namely costly investment and unknown demand, output grows gradually, until demand is met. The present paper extends this theoretical approach to the case where demand is not only unknown, but changes continually. This extension changes the results and the model now generates cycles. This result holds irrespective of the sign of drift of the random walk.

The paper also explores how informational cycles are related to economic growth and to technical change. Introduction of technological innovations, or of new goods, changes supply and demand in unknown ways, since it opens unexplored territories before

1. Zeira (1987) examines the case of a single monopolistic firm, while Rob (1991) analyses a perfectly competitive market. Caplin and Leahy (1993) extend the analysis to a continuous- time framework.

markets. These changes can be uncovered only by producing more. Hence, the missing information of this model is inherently related to growth and technical progress.[2]

Although the basic idea of the paper is presented in a one-sector partial equilibrium model with demand shocks, it is later shown that the main result holds in a general equilibrium version of the model with productivity shocks. This demonstrates the robustness of the basic result. This extension of the model can also be viewed as a business cycle model, as it generates cycles of aggregate output. In the vast theoretical literature on business cycles, this paper is most closely related to real business cycle models, as both describe propagation of uncorrelated productivity shocks into cycles.[3] But despite this similarity, the two approaches are quite different, as shown in Section 6.[4]

The paper is organized as follows. Section 2 describes the basic model. Section 3 derives the dynamic equilibrium and Section 4 demonstrates the cyclicality of output and investment. Section 5 describes the relationship between growth and informational cycles, and Section 6 extends the model to a general equilibrium framework. Section 7 summarizes the paper and the Appendix contains mathematical proofs.

2. THE MODEL

Consider a market for a single good X, which is a non-durable good.[5] Time is assumed to be discrete. The demand for X in period t is described by the following step function D_t:

$$D_t(p_t)=\begin{cases} X_t & p_t \leqq 1 \\ 0 & 1<p_t, \end{cases} \tag{1}$$

where p_t is the price of X in terms of another good Y, which is a numeraire. The maximum amount demanded, X_t, is what we called "potential demand". It is further assumed that potential demand is a random walk, namely that X_t changes according to:

$$X_t = X_{t-1} + v_t, \tag{2}$$

where t_t is random. Changes in demand v_t are not directly observed. It is assumed that $\{v_t\}_t$ are independent, identically distributed random variables, which are not necessarily mean zero, but are bounded and have no mass points. Formally, all random variables v_t have the same density function, f, and there are w and u, $w<u$, such that f is zero outside $[w, u]$ and strictly positive and finite within $[w, u]$. The distribution function of changes in demand, which corresponds to the density function f, is denoted by F. The mathematical expectation of v_t is g:

$$g=\int_w^u vf(v)dv, \tag{3}$$

which is the average change in potential demand.

We now describe the supply side of the market. Each firm produces one unit of the good during one period only. Variable costs of production are c, where: $0<c<1$, in terms

2. A related extension of the model is to immigration to a growing economy. If the capacity of the absorbing country is unknown and increasing, due to economic growth, immigration to this country is characterized by waves, according to our model.

3. Real business cycle models originated in Kydland and Prescott (1982) and Long and Plosser (1983).

4. Informational cycles also differ substantially from other recent business cycle theories, such as endogenous cycles and multiple equilibria, as in Day (1982), Grandmont (1985), Diamond and Fudenberg (1989), Murphy, Shleifer and Vishny (1989), Shleifer (1986) and others.

5. The case of a durable good is discussed in Section 5.

of the numeraire good Y. Investment is made one period ahead of time and costs e units of Y per firm. After one period capital depreciates, whether production occurs or not.[6] Firms are assumed to maximize their intertemporal value. Let K_t be the number of firms created in period $t-1$ in order to produce X in period t. The supply, S_t, is therefore:

$$S_t(p_t)=\begin{cases} K_t & p_t \geqq c \\ 0 & p_t < c. \end{cases} \tag{4}$$

It is further assumed that the market for X is perfectly competitive. Let Y_t denote actual output of the good X in period t. Output is constrained either by previously invested capital, or by demand. Hence:

$$Y_t = \min\{K_t, X_t\}. \tag{5}$$

The informational assumptions of the model are as follows. Potential demand, X_t, is unknown, as are changes in potential demand, v_t. Aggregate gross investment, K_t, is fully known already in period $t-1$, when installed.[7] In period t actual output, Y_t, becomes fully known. Hence, in case of over-investment, when capacity exceeds demand, X_t is revealed as well. In other words, potential demand and the demand curve in general are revealed only by investment and by expansion of production. It is further assumed, for the sake of simplicity, that when investment is made in period t, the market outcome of this period is already fully known to investors. The distribution of demand changes, F, is also known.

Expectations are assumed to be rational, namely entrepreneurs use all the information they have, past and present, to form the best prediction on demand. Their information set consists of past amounts of investment $\{K_s\}_{s\leqq t}$, of past and present amounts of output $\{Y_x\}_{s\leqq t}$ and hence of those past potential demands, X_s, which have been revealed. This information is used optimally, by Bayesian inference, to form the best prediction on the next period's potential demand.

As for the rest of the economy, the credit market is assumed to be perfectly competitive, and agents can borrow and lend any amount at interest rate r, which is assumed to be constant over time. There is free entry into the market one period ahead of time. It is also assumed that investment is profitable when the price of X equals 1 with certainty:

$$\frac{1-c}{1+r} > e, \tag{6}$$

in order to avoid the null case of no investment.

3. EQUILIBRIUM

We first examine temporal equilibrium in the market for X in period t. Demand and supply and market equilibrium are shown in Figure 1. This equilibrium satisfies:

(a) If $K_t \leqq X_t$, output Y_t equals K_t, the price p_t equals 1 and profits per firm equal $1-c$. This case is shown in Figure 1(a).

(b) If $K_t > X_t$, output equals potential demand X_t, the price p_t equals c and profits are zero. This case is shown in Figure 1(b).

6. Full depreciation of capital represents the cost of investment in this model. Partial depreciation could be assumed as well, as in Zeira (1987) and Rob (1991), without any qualitative change in the results.

7. Note that we measure investment by the number of firms, to simplify notation. The cost of gross investment is therefore eK_t.

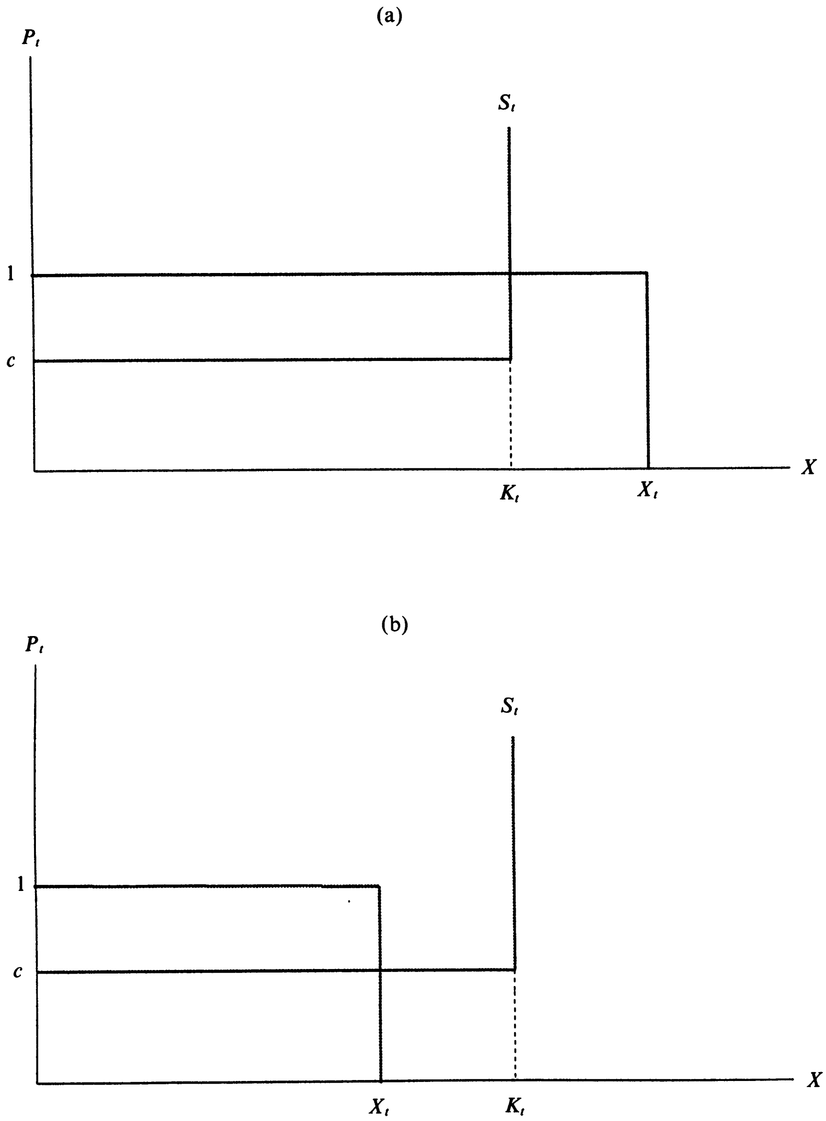

FIGURE 1

We now define intertemporal equilibrium in the market for X.

Definition. An intertemporal equilibrium in the market for X, from period T on, consists of a sequence of distribution functions $\{F_t\}_{t \geqq T}$ and a sequence of positive numbers $\{K_t\}_{t>T}$, such that:

(a) For all $t>T$, F_{t-1} is the distribution function of potential demand X_t, given all the information available in period $t-1$.

(b) For all $t>T$, K_t is the maximum amount of firms with non-negative expected value in $t-1$.

We now recursively construct this intertemporal equilibrium. According to the above description of market outcomes in t, the net expected value of investment in $t-1$ is:

$$\frac{1}{1+r}[(1-c)P_{t-1}(X_t \geqq K_t)]-e, \tag{7}$$

where P_{t-1} is probability in $t-1$, based on the distribution F_{t-1}. Due to free entry, investors enter in period $t-1$ as long as the net expected value is greater than or equal to zero. Hence, gross investment in period $t-1$ is:

$$K_t = \max\left\{K: P_{t-1}(X_t \geqq K_t) \geqq \frac{e(1+r)}{1-c}\right\} = \max\{K: P_{t-1}(X_t < K) \leqq a\}, \tag{8}$$

where:

$$a = 1 - \frac{(1+r)e}{1-c}, \tag{9}$$

and according to equation (6): $0 < a < 1$.

Lemma 1. *K_t always exists. Furthermore, $K_t = \sup\{K: F_{t-1}(K) \leqq a\}$, and, if F_{t-1} is continuous, $F_{t-1}(K_t) = a$.*

Proof. See Appendix. ||

We next show how investors use F_{t-1} and the new information revealed by the market in period t to form the next distribution function F_t. This is done in two steps:

1. First F_{t-1} is updated to G_t as follows:
   (a) If potential demand is met and $X_t < K_t$, then X_t is known and:

$$G_t(x) = \begin{cases} 0 & x < X_t \\ 1 & x \geqq X_t. \end{cases} \tag{10}$$

   (b) If demand has not yet been reached and $K_t \leqq X_t$, X_t is yet unknown, but entrepreneurs know that it is greater than or equal to K_t. Hence, the Bayesian updated distribution is:

$$G_t(x) = \begin{cases} 0 & x < K_t \\ \dfrac{F_{t-1}(x) - P_{t-1}(X_t < K_t)}{P_{t-1}(X_t \geqq K_t)} & x \geqq K_t. \end{cases} \tag{11}$$

   Note, that if F_{t-1} is continuous:

$$G_t(x) = \begin{cases} 0 & x < K_t \\ \dfrac{F_{t-1}(x) - a}{1-a} & x \geqq K_t. \end{cases} \tag{12}$$

2. In the second step G_t, equation (2) and the independence of X_t and v_{t+1} are combined to derive the distribution function F_t of X_{t+1}:

$$F_t(x) = \int_w^u G_t(x-v)f(v)dv, \quad \text{for all } x. \tag{13}$$

Lemma 2. *For any distribution function F_{t-1}, the next-period distribution function, F_t, is continuous.*

Proof. See Appendix. ||

We can now fully prove the existence and uniqueness of intertemporal equilibrium.

Proposition 1. *For any initial distribution function F_T, there exists a unique intertemporal equilibrium, which begins with F_T and which satisfies:*
(a) *F_t is continuous for all $t>T$*
(b) *$F_{t-1}(K_t)=a$, for all $t>T+1$.*

Proof. According to Lemma 1, K_t exists and is unique for any distribution function. The rest of the proof is by mathematical induction. If the distribution F_{t-1} embodies all the information known in period $t-1$, and if it determines F_t by using period t market outcome, as shown in equations (10)–(13), then F_t embodies all the information available in period t. Note also, that F_t is continuous according to Lemma 2. This completes the proof of the proposition. ||

Due to full depreciation in this model, gross investment in period t equals K_{t+1}. Operating capital in period t equals Y_t, which is not always equal to K_t. Hence, net investment in period t, I_t, equals $K_{t+1}-Y_t$. We will use the terms 'net investment' and 'investment' interchangeably in the rest of the paper.

We now turn to define cycles in this model. From the above it is clear that the market alternates between periods of excess demand and periods of excess capital. We want to define a cycle as an interval time, during which the market hits potential demand and then goes through excess demand, until it hits potential demand again, and a new cycle begins.

Definition. If potential demand is reached once in t_0 and again in t_1, but not in between, namely: $K_t>X_t$ in $t=t_0$ and in $t=t_1$, and if $K_t \leqq X_t$ for $t_0<t<t_1$, then the time interval $t_0, \ldots, t_1-1$ is called an informational cycle.

According to this definition, the dynamic equilibrium is divided to a sequence of consecutive cycles. A cycle begins when demand is reached and ends just before it is reached again. For the term cycles to be more meaningful than merely a division into sub-intervals, we need to show that output and investment fall and rise in each cycle. Furthermore, we need to show that informational cycles occur repeatedly, namely that cycle duration is finite. All this is shown in the next section.

4. DESCRIPTION OF INFORMATIONAL CYCLES

In this section we describe informational cycles: their similarities, their lengths, and the behaviour of output and investment during a cycle. We first claim that although cycles differ with respect to duration, all cycles are similar, while they last. This claim is formalized in Proposition 2.

Proposition 2. *There exists a sequence of distribution functions,* $\{H_t\}_{t\geqq 1}$ *and a sequence of investment levels* $\{i_t\}_{t\geqq 1}$*, such that each cycle satisfies:* $F_{T+t-1}(x) = H_t(x - X_T)$*, for all* x*, and* $I_{T+t-1} = i_t$*, where* T *is the first period in the cycle and* $t = 1, 2, \ldots$ *as long as the cycle continues.*

Proof. See Appendix. ||

The basic intuition behind Proposition 2 is that changes in demand are i.i.d. and hence future prospects look similar, once potential demand is reached. From Proposition 2 it also follows that as long as the cycle continues, investment and output growth are deterministic.

Notice that the sequence $\{i_t\}_t$ from Proposition 2 can also be defined by the following condition:

$$P\left(\textstyle\sum_{j=1}^{t} v_j \geqq \sum_{j=1}^{t} i_j \mid \sum_{j=1}^{s} v_j \geqq \sum_{j=1}^{s} i_j, \forall s < t\right) = 1 - a. \tag{14}$$

We next turn to examine net investment and output growth along a typical cycle, by analysing the sequence $\{i_t\}$. According to Proposition 2 this sequence depends only on the density function f and on the parameter a. We first examine net investment in the first period, i_1:

$$\int_{w}^{i_1} f(v)dv = a, \tag{15}$$

Hence, if a is small, investment is small as well and is close to the lower bound of demand change w. Investors are hesitant, as they know that demand cannot be very large.

But after cautious investment in the first period of the cycle investment rises, as changes in demand open up new opportunities. We show this using the following two propositions.

Proposition 3. *Net investment* $\{i_t\}_t$ *satisfies:* $i_t > i_1$*, for all* $t > 1$.

Proof. See Appendix. ||

Can we make a broader claim, namely, that net investment $\{i_t\}$ always increases? This is not generally the case, as can be shown by a counter-example, but there are many density functions f for which this sequence is indeed increasing. Figure 2 shows net investment in the case of a uniform distribution between 0 and 1 and for various values of a. As the figure shows, net investment is increasing in this case. Furthermore, simulations show that net investment is increasing for many other distributions. What we can prove in the most general case is that average investment rises, so that at some point in time it exceeds the average growth of potential demand.

Proposition 4. *For any* $h < g$*, where* g *is average change of demand, there exists an integer* N*, such that:* $(\sum_{t=1}^{n} i_t)/n > h$*, for all* $n \geqq N$.

Proof. See Appendix. ||

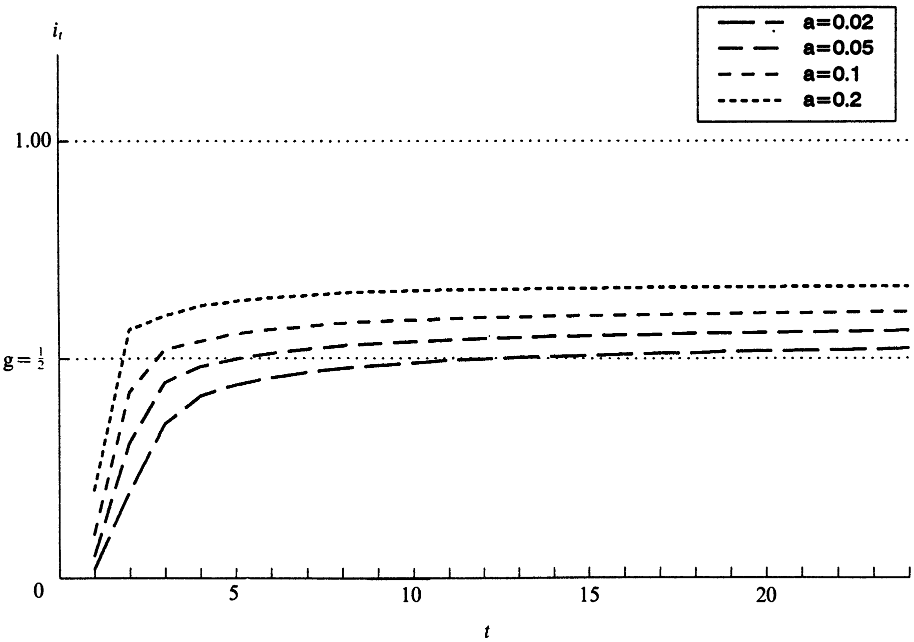

FIGURE 2

Corollary. *liminf* $(\sum_{t=1}^{n} i_t)/n \geqq g$.

We now turn to examine the duration distribution of cycles.

Proposition 5. *The duration of cycles is geometrically distributed: the probability of a cycle lasting exactly t periods is* $a(1-a)^{t-1}$. *Therefore:*

(a) *Each cycle is of finite duration with probability one.*
(b) *The average length of cycles is* $1/a$; *it depends positively on investment costs e and on the interest rate r and negatively on profitability* $1-c$.

Proof. See Appendix. ||

Note, with respect to Proposition 5, the following points:

(a) Proposition 5 shows that if the market hits potential demand once, it will hit it again with probability 1. By the same token, it can be shown, that from any initial distribution F_T, the market hits potential demand with probability 1.
(b) The intuitive explanation for (b) in Proposition 5 is that variables causing a reduction in the rate of investment extend informational cycles, while variables that increase investment curtail them. Thus, high profitability raises cycles' frequency and high interest lowers it.[8]
(c) The duration distribution of cycles depends on a only and is independent of the distribution of v_t. Hence, frequency of cycles a is independent of their amplitude.[9]

8. One empirical implication of this result is that sectoral cyclical frequency should be positively correlated with sectoral profitability.

9. This result seems to be rather model specific. Other specifications of the theory may yield different results.

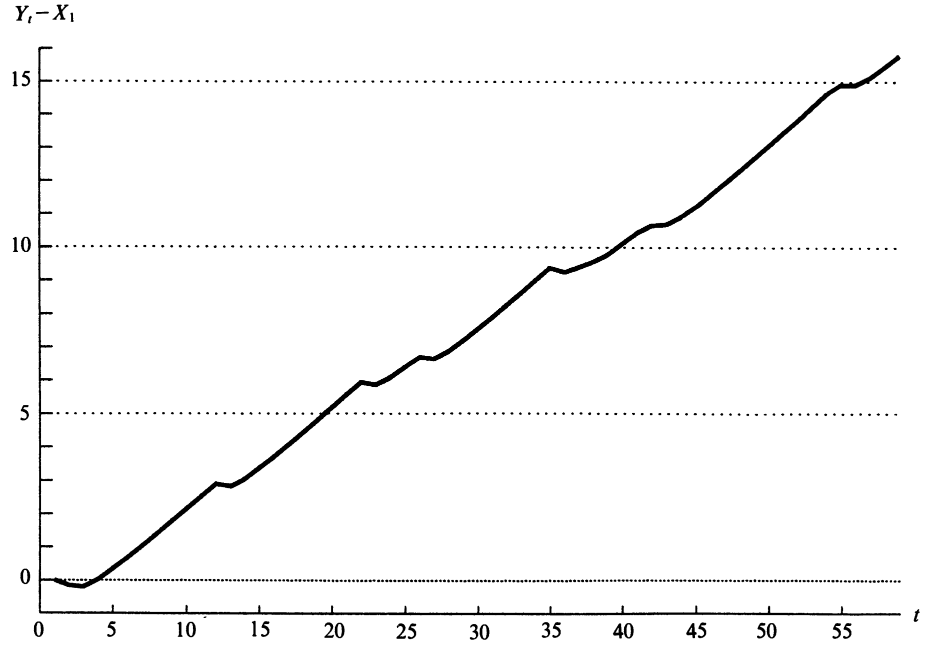

FIGURE 3

(d) If cycle duration is finite, output must increase by more than potential demand in order to reach it. Hence, average net investment must at some point strictly exceed g. This conclusion is slightly stronger than Proposition 3.[10]

We can summarize the results of this section as follows: output and investment follow a cyclical pattern. Every once in a while, output hits potential demand, whereupon investment and output growth fall. Then investment gradually accelerates and even exceeds the average growth of potential demand, until output hits potential demand again and a new cycle begins. Figure 3 shows a typical time plot of output (or more precisely, output Y_t minus initial output X_1), for the case of demand growth which is uniformly distributed between $-0 \cdot 2$ and $0 \cdot 8$, and for: $a = 0 \cdot 1$. Figure 3 demonstrates the cyclical pattern of output and investment.

5. GROWTH AND INFORMATIONAL CYCLES

Our model shows that informational cycles emerge when demand behaves like a random walk. This result holds for any sign of the drift of this random walk. The most interesting case though, is that of growing demand, both because it is a more realistic case and because informational cycles are inherently related to economic growth.

In a growing economy innovations continuously raise productivity, income and demand, and new goods are continuously being invented.[11] Hence demand grows, but no one knows by how much until it is revealed by the market, because these innovations are

10. Indeed, Zeira (1992) shows that if investment is increasing, it strictly exceeds g at some point.

11. The relationship between growth and technical change has been thoroughly discussed in the literature, as can be seen in Link (1987). Recent literature on endogenous growth also emphasizes this relationship. See Romer (1990), Stokey (1988) or Grossman and Helpman (1991).

experienced for the first time. No one knows in advance how productive a new technology is, or how popular a new product will be, before it is tested by the market through actual production. Hence, the missing information, which is described in this paper, is inherently related to growth.

In order to examine informational cycles when demand grows, we consider two cases: the weak case, in which demand grows on average ($g>0$), and the strong case, in which demand always grows ($v_t \geqq 0$, or: $w=0$). In both cases output and investment are cyclical, as shown in previous sections, but cycles in the two cases are slightly different. In the weak case, demand can sometimes fall, since $w<0$, and hence output also falls at the beginning of the cycle, if a is small. Hence, in the weak case output grows on average, but it declines at the beginning of each cycle. In the strong case of demand growth, output always grows, though at varying rates. But if we change the model slightly, we can have informational cycles in which output falls in the beginning of cycle, even in the strong case of growth, when demand always rises.[12]

In the benchmark model X is non-durable. We now show that if this assumption is removed, output falls in the beginning of each cycle.[13] Consider the same model as in Section 2, except that X is durable. If 1 unit of X is stored, d units remain after one period, where $0<d\leqq 1$. For the sake of simplicity, we assume in this case that $c=0$. Hence, firms are indifferent between selling X and storing it at a price $p=ed$. Hence the equilibrium price of X is 1 if capacity is not met, and ed if capacity is reached. Let us define a new variable S_t, the quantity supplied of $X: S_t=K_t+dh_{t-1}$, where h_{t-1} is the amount of goods unsold in period $t-1$. The condition that determines equilibrium investment in period $t-1$ is therefore:

$$\frac{1}{1+r}[1-F_{t-1}(S_t)+edF_{t-1}(S_t)]=e, \tag{16}$$

or $F_{t-1}(S_t)=b$, where $b=[1-e(1+r)]/[1-ed]$. Hence, the dynamics of S_t are the same as in the basic model (except that b replaces a) and are, therefore, cyclical as well. As long as a cycle goes on, S_t equals K_t. But when potential demand is reached and a new cycle begins in period T, firms are left with excess production h_T and net investment i_1-h_T is negative, if b is small. Hence, output declines in the following period. Intuitively, if the good is durable and production exceeds demand, inventories increase and that reduces production.

6. INFORMATIONAL BUSINESS CYCLES

The benchmark model of informational cycles described in the previous sections is a partial equilibrium model, driven by demand changes. In this section we construct a general equilibrium version of the model, where productivity changes generate similar informational cycles. This extension of the model serves three purposes. First, it shows that informational cycles are consistent with a general equilibrium framework and can also be welfare evaluated. Second, it shows the robustness of the theory, as it holds not only for demand shocks, but for productivity shocks as well. Third, it shows how the model can explain cycles of aggregate output and thus add to our understanding of business cycles.

12. Notice that this shows that the informational cycles model is not subject to the critique often raised against real business cycle models, in which recessions are caused by severe adverse shocks, which are hard to observe in reality. See Mankiw (1989).

13. A similar result holds if the assumption of full capital depreciation is removed.

Consider an economy which produces three goods: X, Y and Z. The good X is an intermediate good and is produced by capital only. Y is a final good, used both for consumption and investment, and produced by labour and the intermediate good X. There is an additional good Z, which is produced at home by labour only, for self-consumption, and is not marketable. We assume that workers differ with respect to their productivities. A worker can produce either amount w of the good Z or amount $2w$ of Y, if he uses amount w of the intermediate good X. The price of the intermediate good X, in terms of the final good Y, is p. The intermediate good X is produced by capital, where e units of capital (namely of Y) must be invested in period $t-1$, in order to produce 1 unit of X in period t. For simplicity assume that there are no additional costs of production in period t.

Individuals in this economy live infinitely and each supplies one unit of labour in every period. Individuals are risk neutral and their utility is described by:

$$U=\sum_{t=0}^{\infty}(1+r)^{-t}c_t, \tag{17}$$

where c_t is consumption of either Y or Z in period t. We therefore assume that individuals are indifferent between consumption of these two goods and consume the one which costs less, or consume Y, if they cost the same. Hence, as long as the price p of the intermediate good X is less than or equal to 1, each individual produces Y and demands amount w of the intermediate good. The subjective discount rate r equals the interest rate in every period, due to risk neutrality. There is a continuum of individuals of size 1. Individuals are heterogenous with respect to their productivities and to how their productivities change as new technologies are introduced to the economy. Let w_j^t be the productivity of individual j in period t. Aggregate productivity in period t is:

$$X_t=\int_0^1 w_j^t dj. \tag{18}$$

It is easy to see that demand for X is described by the same demand function D_t in equation (1), with X_t from equation (18).

Next assume that in each period a technological innovation arrives, which changes aggregate productivity by v_t, so that equation (2) is satisfied, and assume that $\{v_t\}$ satisfy all the assumptions of Section 2. We further assume that aggregate productivity and changes in this productivity are not directly observed. This is possible even if productivity at the individual level is observable, since individuals are heterogenous with respect to their productivity and its change. Similarly, assume that aggregate production of Z is not observable, since it is home production.

In the economy described here, the market for the intermediate good X behaves exactly as described in Section 2. Hence, the general equilibrium of this economy replicates the dynamic equilibrium described in Sections 3 and 4 and investment and output of X are cyclical. As a result, aggregate output of Y is cyclical as well. Hence, our theory can explain aggregate cyclical fluctuations namely business cycles.

Informational business cycles are similar in many respects to real business cycles. Both are driven by productivity shocks, which are uncorrelated and are propagated into cycles. But there are also important differences between the two types of cycles. First, the propagation mechanism in the present model is informational. Second, unlike real business cycles, informational cycles are not Pareto-optimal. This is due to an informational externality, which was first described by Rob (1991). Investors who enter the market in period t, generate not only direct profits, but also information, which is freely available to later entrants. Due to this externality, equilibrium is not optimal. It can be shown that optimal investment should be more rapid, and therefore optimal cycles should be shorter and more frequent, than in free competition. Another difference between informational cycles and real business cycles, is in the shape of cycles. Informational cycles can be described as a Markov process, with given transition probabilities from boom to recession, rather than an autoregressive process, as real business cycles are.[14]

7. FINAL COMMENTS

This paper claims that missing aggregate market information, and the search for this information, can be part of the explanation of cyclical movements in output and investment. The paper shows that if the maximum amount that can be produced or sold is unknown, and is continually changing, then firms enter the market in waves, and hence output and investment are cyclical. These waves are explained by two forces. One is the cost of over-investment, which slows entry to the market; the other is the creation of new profitable opportunities, which encourage entry and investment. At the beginning of the cycle, immediately after demand or production constraints are met, the second force is weak, and thus entry is slow. As the cycle continues, the second force becomes stronger, overcomes the first, and drives the market into new constraints. The evolution of information plays an important role in these dynamics, and hence we call these cycles "informational".

APPENDIX

Proof of Lemma 1. Consider the set $A=\{K:P_{t-1}(X_t<K)\leqq a\}$. This set is bounded from above and hence has a finite supremum, K'. There is, therefore, a sequence $\{K^n\}$ in A, which converges to K' from below. Hence:

$$P_{t-1}(X_t<K')=\lim_{n\to\infty} \mathrm{P}_{t-1}(X_t<K^n)\leqq a,$$

and therefore K' is in A as well. Hence, K' is a maximum and is equal to K_t. The rest of the proof is straightforward. ||

Proof of Lemma 2. This lemma is a special case of a more general claim: if v_{t+1} has no mass and is independent of X_t, then $X_{t+1}=X_t+v_{t+1}$ has no mass either.

The proof goes as follows: let $x>y$, then:

$$F_t(x)-F_t(y)=\int_w^u [G_t(x-v)-G_t(y-v)]f(v)dv.$$

As y approaches x, $G_t(x-v)-G_t(y-v)$ converges to zero except on a countable set of points, since G_t is a distribution function. Hence, the above integral converges to zero as y approaches x and F_t is continuous. ||

Proof of Proposition 2. We prove the proposition by mathematical induction. Define first the distribution H_1 and investment i_1 as follows: $H_1=F$ and $F(i_1)=a$. When the cycle starts in period T, we have:

$$F_T(x)=\int_{-\infty}^{x-X_T} f(v)dv=F(x-X_T),$$

according to equations (10) and (13). Hence: $F_T(x)=H_1(x-X_T)$, for all x. Note too, that: $a=F_T(K_{T+1})=F(K_{T+1}-X_T)=F(I_T)$. Hence: $I_T=i$.

Assume now that the proposition holds for all $s\leqq t$. Define H_{t+1} in two stages. First:

14. Many recent empirical works analyse cycles as such Markov processes, among them Neftcy (1982), Hamilton (1989), Diebold and Rudenbusch (1990, 1991) and others.

$$\bar{H}_t(x)=\begin{cases} 0 & x\leqq\sum_{s=1}^{t} i_s \\ \dfrac{H_t(x)-a}{1-a} & x>\sum_{s=1}^{t} i_s, \end{cases}$$

and then:

$$H_{t+1}(x)=\int_{-\infty}^{\infty}\bar{H}_t(x-v)f(v)dv,$$

for all x. Investment in period $t+1$, i_{t+1}, is defined by:

$$H_{t+1}(\textstyle\sum_{s=1}^{t} i_s+i_{t+1})=a.$$

It is easy to verify that: $F_{T+t}(x)=H_{t+1}(x-X_T)$ for all x, and that: $I_{T+t}=i_{t+1}$. ||

Proof of Proposition 3. According to the proof of Proposition 2, for each $t\geqq 1$ we have:

$$a=H_{t+1}(\textstyle\sum_{s=1}^{t+1} i_s)=\displaystyle\int_{-\infty}^{i_{t+1}}\bar{H}_t(\textstyle\sum_{s=1}^{t+1} i_s-v)f(v)dv<\displaystyle\int_{-\infty}^{i_{t+1}}f(v)dv=H_1(i_{t+1}).$$

Therefore: $i_{t+1}>i_1$, for all $t\geqq 1$. ||

Proof of Proposition 4. For every n:

$$P(\textstyle\sum_1^n v_t\geqq\sum_1^n i_t)=P(\sum_1^n v_t\geqq\sum_1^n i_t|A)\,P(A)+P(\sum_1^n v_t\geqq\sum_1^n i_t|A^c)\,P(A^c),$$

where A_t is the event

$$\textstyle\sum_1^s v_t\geqq\sum_1^s i_t,\quad \text{for all } s<n.$$

Since:

$$P(\textstyle\sum_1^n v_t\geqq\sum_1^n i_t|A^c)\leqq P(\sum_1^n v_t\geqq\sum_1^n i_t|A),$$

we get:

$$P(\textstyle\sum_{t=1}^n v_t\geqq\sum_{t=1}^n i_t)\leqq P(\sum_{t=1}^n v_t\geqq\sum_{t=1}^n i_t|\sum_{t=1}^s v_t\geqq\sum_{t=1}^s i_t,\ \forall s<n)=1-a.$$

Let us now assume that the proposition does not hold, so that there is a positive number $h<g$ and a subsequence $\{n_j\}$, for which:

$$\frac{1}{n_j}\textstyle\sum_{t=1}^{n_j} i_t\leqq h,\quad \text{for all } j.$$

We can therefore apply the above inequality and get:

$$1-a\geqq P(\textstyle\sum_{t=1}^{n_j} v_t\geqq\sum_{t=1}^{n_j} i_t)\geqq P\left(\dfrac{1}{n_j}\sum_{t=1}^{n_j} v_t\geqq h\right).$$

Since the right-hand term in this equation converges to 1, according to the weak law of large numbers, we get a contradiction. Hence the proposition holds. ||

Proof of Proposition 5. The probability that a cycle lasts only one period is a, the probability that potential demand is reached in the first period of the cycle. Hence, the probability of a cycle lasting more than 1 period is $1-a$. Let us denote by Q_t the probability of a cycle lasting at least t periods. Then, $Q_1=1-a$, and generally:

$$Q_t=P(\textstyle\sum_{j=1}^s v_j\geqq\sum_{j=1}^s i_j,\ \forall s\leqq t).$$

According to equation (14) $Q_t/Q_{t-1}=1-a$ and hence we get by induction that $Q_t=(1-a)^t$. Let us denote by R_t the probability of the cycle lasting exactly t periods. R_t, therefore, equals:

$$R_t=P(\textstyle\sum_{j=1}^{t} v_j<\sum_{j=1}^{t} i_j, \sum_{j=1}^{s} v_j\geqq\sum_{j=1}^{s} i_j, \forall s<t)$$

$$=P(\textstyle\sum_{j=1}^{t} v_j<\sum_{j=1}^{t} i_j|\sum_{j=1}^{s} v_j\geqq\sum_{j=1}^{s} i_j, \forall s<t)\, Q_{t-1}$$

$$=aQ_{t-1}=a(1-a)^{t-1}.$$

Hence, the length of cycles is geometrically distributed.

The probability that a cycle is infinite must be less than Q_t for any t, and is therefore 0. This proves part (a) of the theorem.

Given the distribution of cycle duration, the average length of a cycle is:

$$\textstyle\sum_{t=1}^{\infty} ta(1-a)^{t-1}=\frac{1}{a}=\frac{1-c}{1-c-e(1+r)},$$

and this proves part (b). ||

Acknowledgement. I am grateful to Yali Amit and Sergiu Hart for their generous help. The paper has benefited from the valuable comments of Roland Bénabou, Joseph Djivre, Toni Lancaster, Torsten Persson, Oren Sussman and seminar participants at Brown University, the Hebrew University, Tel-Aviv University, Harvard University, Boston University and Berkeley. I also thank the editors and two anonymous referees for their useful comments and suggestions. All remaining errors are, of course, mine.

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