# Knowing What Matters to Others: Information Selection in Auctions.*

Nina Bobkova†

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**Abstract** The valuation of bidders for an object consists of a common value component (which matters to all bidders) and a private value component (which is relevant only to individual bidders). Bidders select about which of these two components they want to acquire noisy information. Learning about a private component yields independent estimates, whereas learning about a common component leads to correlated information between bidders. I show that in a second price auction, information selection in equilibrium is unique. Bidders only learn about their private component, so an independent private value framework arises endogenously. If this were not the case, a bidder could guarantee himself the same expected gain and a strictly lower payment by decreasing correlation in private information. In an all-pay auction, bidders also prefer information about private components. In a first price auction, increasing correlation strictly elevates the payoff for a bidder under certain conditions.

**JEL codes:** D44; D83.

**Keywords:** Information selection; endogenous interdependence; affiliated signals; auctions.

# 1 Introduction

Bidding preparation for auctions usually involves evaluating multiple characteristics. This paper delves into which characteristics bidders should gather information about and how such decision is influenced by the auction format in cases wherein people cannot take into account all existing information.

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†Email: nbobkova@uni-bonn.de. University of Bonn, Adenauerallee 24-42, 53113 Bonn, Germany.

These issues are relevant to, for example, corporate takeovers, in which acquiring firms have access to a variety of information about a target company. This information encompasses the R&D activities and the book value. A reasonable assumption is that firms cannot perfectly process or uncover all existing information, and are thus driven to select elements to focus on before bidding takes place. Should an acquiring firm conduct research on aspects that are specific to them, such as their R&D overlap? Or should they focus on factors that also matter to other acquiring firms, such as the book value of a target?

Another example are resource rights auctions for oil fields or timber. Each bidder derives the same monetary value from an unknown volume of oil or timber on a site, and this value stems from the market price. Bidders may incur different costs in extracting the resources from a site because of the use of different drilling or logging technologies and variances in experience levels. I inquire into whether a bidder prefers to perform an exploratory drilling to learn about oil volume (i.e., the common component) or to learn about the costs of extracting the resource through estimations of the drilling costs specific to him (i.e., the private component).

Buying real estate is yet another case that involves evaluating a variety of attributes prior to bidding. These attributes include the costs of maintaining a property, local taxes, mortgage rates, the convenience of travel to work, and personal preference for a property. Do bidders prefer to acquire information on the qualities of a property that are pertinent to all bidders, such as maintenance costs? Or would they rather examine characteristics that are uniquely related to them, such as the convenience of traveling to work from a property.

The contribution of this paper is to investigate the incentives provided by a variety of auction formats regarding information selection. I demonstrate that bidders prefer to learn about their private components in the second price auction (SPA) which is commonly used in the examples[1] described above. I also analyze incentives to selecting information in a first price auction (FPA) and an all-pay auction.[2]

The novelty of this paper lies in its illumination of *which* random variables bidders seek to learn (*information selection*) instead of what *level of accuracy* of information they favor about a given real-valued random variable (*information acquisition*). I isolate incentives for learning about the signal of an opponent: Holding the level of accuracy constant, do bidders prefer their private information to be dependent (information about a common component) or independent (information about a private component)?

The independent private values setting (IPV) and the interdependent values setting (IntV) lead to different theoretical predictions and vary significantly in their implications for auction design and policy.[3] The literature on auctions usually assumes either IPV or IntV setting at the outset of the analysis. In addition, identifying the valuation setting on the basis of data is often challenging if not

[1]See Porter (1995) for a survey of oil and gas lease auctions and Hendricks and Porter (2014) for an analysis of the auction mechanisms in selling resource rights in the U.S. See Gentry and Stroup (2017) for an analysis of auctions and negotiation procedures commonly used in mergers and acquisition, and Chow and Ooi (2014) for real estate land auctions.

[2]As I concentrate on the case of two bidders, my results also hold for the open English auction (equivalent to the SPA) and the Dutch auction (equivalent to the FPA) (Milgrom and Weber, 1982).

[3]In the IPV setup, each bidder's private information matters only to him; in the IntV setup, a bidder's estimate of the object depends on the private information of other bidders.

impossible.[4] By restricting the ability of bidders to learn about more than one attribute, I study which value setting arises endogenously.

For a brief outline of the model, consider two bidders who compete for one indivisible object in a SPA. They share the same common component (e.g., the book value of a firm) and have independent private value components (e.g., match-specific R&D overlap). The valuation of each bidder is the sum of two value components about which they are uninformed. Bidders select between learning about a common or a private component. Information selection is simultaneous and covert. Considering both components is infeasible.[5]

Learning about the common or the private component has equal accuracy. In a single agent problem, an agent would be indifferent between learning about either component, as the two experiments are equally informative about the total value of the object. Yet, in the strategic context of an auction information about the object plays a dual role. Beyond containing information about the object's worth, it is also informative about the signal of the opponent and his bid. Moreover, a rational bidder conditions his estimate of the object not only on his own information, but also on what he learns from the event of winning. Being the highest bidder when the opponent learns about the common component implies that the opponent has a low signal realization. This is bad news for the expected value of the object. In equilibrium, a bidder shades down his bid due to this so-called winner's curse.

In my model, the extent of the winner's curse and the interdependence between bidders' information are endogenous and depend on which value component bidders learn about. The signals of bidders become more affiliated if they learn about the common component. The winner's curse exacerbates. If other bidders learn only about their private component, their information bears no relevance for other bidders and there is no winner's curse. Two standard valuation settings are nested in my model. An IPV setting arises if both bidders learn only about their private components. A pure common value setting emerges if both bidders learn only about the same common component.

The result for the SPA with two bidders is that in any symmetric equilibrium, information selection is unique: There is only learning about the private component, and an IPV setting arises endogenously. The SPA induces the ex-ante efficient outcome. No resources are wasted by learning about the common component which is irrelevant for efficiency, and the object is allocated to the bidder with the highest estimate of his private component. This result holds in a general class of utility functions.

In the SPA, a bidder could always find a strictly profitable deviation by decreasing interdependence in private signals. The approach is to find a deviation strategy that keeps the expected gain and winning probability constant, while strictly decreasing the expected payment. For a sketch of the argument, consider a candidate equilibrium in which both bidders learn only about the common component and have the highest degree of interdependence in private signals. Then, the following deviation is strictly profitable for a bidder: *Learn about the private component, but bid as if it were a signal about the common component.* This strategy eliminates interdependence in private signals but employs the same

[4]See Laffont and Vuong (1996) for a general discussion of identifying the value setting in the FPA, and Athey and Levin (2001) for timber auctions.

[5]Learning about a component might involve some actuarial calculations or an experiment, e.g., exploratory drilling. I analyze a scenario where such an experiment is non-divisible, and analyzing both components half-way does not produce meaningful information: drilling half a hole, or calculating only the first half of a cost-benefit analysis is not useful.

bidding function as the candidate equilibrium for tractability.

The expected payment conditional on winning from such a deviation strategy is strictly lower. The higher the interdependence, the higher the distribution of the second order statistic of the opponent's signal and his bid. By decreasing interdependence, the distribution of the second order statistic puts more weight on lower bids, and expected payment strictly decreases.

The expected gain from this deviation is the same as in the candidate equilibrium. For every realization of the total value of the object, the probability of placing the highest bid is the same with the candidate equilibrium and the deviation strategy. However, given a total value for the object, winning probability for different compositions of the two components changes with deviating. In the candidate equilibrium, as both bidders learn about the common component, they win with equal probability for each realization of it. In the deviation strategy, a deviating bidder is more likely to win in states that involve a high private and a low common component, and vice versa. The existence of a deviation strategy that leads to the same expected gain for a strictly lower payment pushes the incentives of bidders in the SPA towards independence, and yields a unique information choice in equilibrium of the SPA.

In a FPA, incentives to select information are opposite to the SPA. As a winning bidder pays his own bid, he does not want to "leave money on the table" by overbidding his opponent by too much. Having a better estimate of the opponent's bid can reduce the expected payment conditional on a win as it reduces the first order statistic of winning bids. Therefore, increasing the dependence of the own signal with the signal of the opponent induces a lower payment if both follow the same bidding function. I show that the ex-ante efficient equilibrium in which bidders learn only about their private component is not robust: After introducing a slight degree of correlation between the common and the private component, bidders prefer more interdependence by learning about the common component.

In addition, I consider information selection in all-pay auctions. For the general case of many bidders, bidders learn only about the private component and an IPV setting arises endogenously. This is because by deviating to the private component, bidders can always guarantee themselves a weakly higher winning probability at every total value of the object, for the same expected payment.

Section 1.1 describes the related literature. Section 2 introduces the model and the informational framework. The analysis in Section 3 shows the consequences of information selection on the joint signal distributions (Section 3.1) and on the value of the object (Section 3.2). I combine those observations in Section 4 to solve for an equilibrium of the SPA. Then, I show that the results generalize to a broader class of utility functions in Section 5.1, and discuss the effect of more than two bidders in Section 5.2. Finally, I analyze the FPA in Section 6.1, and the all-pay auction in Section 6.2.

## 1.1 Related Literature

In the classic literature in Auction Theory, the distribution of private information of bidders is exogenous and does not depend of the choice of the auction format.[6] In their seminal work, Milgrom and Weber (1982) introduce a theory of affiliation in signals, and derive the equilibrium for the SPA, the

[6]For an IPV setup, see Vickrey (1961) and Riley and Samuelson (1981). For a common value model, see Wilson (1969) and Milgrom (1981), and Milgrom and Weber (1982) for a general interdependent setup with affiliated signals.

FPA and English auction. The all-pay auction for affiliated signals has been analyzed by Krishna and Morgan (1997) and recently by Chi et al. (2017).

The literature on information acquisition in auctions[7] endogenizes the private information of bidders, by asking *how much* costly information they seek to acquire.[8] Bidders choose the informativeness of their signal about a single-dimensional payoff relevant variable, usually for a fee that increases in the amount of information it contains.[9]

Persico (2000) considers costly information acquisition in an interdependent value model in the FPA and the SPA. Before bidding, bidders choose the *accuracy* of their signal about a one-dimensional random variable. Accuracy is a statistical order on the informativeness of an experiment by Lehmann (1988).[10] In the model of Persico (2000), learning with higher accuracy has two effects: first, the information about the own valuation becomes more precise; second, bidders obtain a better estimate of the signals of other bidders. Therefore, a higher accuracy inextricably links these two effects. Persico (2000) shows that incentives for information acquisition are stronger in the FPA than in the SPA.

In contrast to Persico (2000), my model fixes the effect of informativeness about the object, and concentrates on choosing more or less correlation with the opponent. In my model, there are two signals available about two payoff-relevant variables. Accuracy of information is fixed and equal in each available signal. In contrast to Persico (2000), bidders in my model have to select the variable about which they prefer to learn. The results in Persico (2000) are of a relative nature: given a level of accuracy acquired in the SPA, the level of accuracy in a FPA is higher.[11] In contrast, my framework provides an absolute prediction: about *which* component do bidders learn.

In Bergemann et al. (2009), the value of an object is a weighted sum of everybody's payoff type. Information acquisition is binary: either learn perfectly about the own payoff-type, or learn nothing. Note that in this formulation, learning cannot introduce any dependence between the signal of bidders, as all payoff types are distributed independently (although they matter to other bidders). With positive interdependencies in payoff types, Bergemann et al. (2009) show that in a generalized Vickrey-Clarke-Groves mechanism[12] bidders acquire more information than would have been socially efficient.

In the IPV setup of Hausch and Li (1991), the SPA and FPA induce equal incentives to acquire

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[7]Endogenous information acquisition has been analyzed in other areas of Economics. E.g., see Bergemann and Välimäki (2002), Crémer et al. (2009), Shi (2012) and Bikhchandani and Obara (2017) in optimal and efficient mechanism design, Martinelli (2006) and Gerardi and Yariv (2007) in committees, Crémer and Khalil (1992) and Szalay (2009) in principal-agent-settings, and Rösler and Szentes (2017) in bilateral trade.

[8]In the context of auctions, information acquisition has been modeled in an IPV model (see e.g. Hausch and Li, 1991; Compte and Jehiel, 2007; Gretschko and Wambach, 2014), and in an IntV framework (see e.g. Persico, 2000; Bergemann et al., 2009).

[9]Informativeness criteria include Blackwell sufficiency (Blackwell, 1951), accuracy (Persico, 2000; Lehmann, 1988), dispersion measures (Ganuza and Penalva, 2010), or deciding whether to learn perfectly or nothing about a payoff relevant variable (e.g. Bergemann et al., 2009). Better informativeness usually comes at higher costs.

[10]The concept of accuracy of of a statistical experiment is established by the name of 'effectiveness' by Lehmann (1988) in the statistical literature.

[11]This holds under appropriate conditions on the marginal costs for increasing accuracy.

[12]See Dasgupta and Maskin (2000) for a generalized Vickrey-Clarke-Groves mechanism in the context of auctions, and Jehiel and Moldovanu (2001) for a general mechanism design setting with externalities in information and allocations.

information about the one-dimensional value. Stegeman (1996), showing that the incentives to acquire information in an IPV setting coincides in FPA and SPA, and with the incentives of a planner, making information acquisition efficient.

The above literature on information acquisition in auctions considers *covert* information acquisition. That is, bidders do not know how much information their competitors acquire before the auction. Another strand of the literature also analyzes *overt* information acquisition, where bidders observe how much information others acquired before bidding. Hausch and Li (1991) show that the SPA and the FPA induce different incentives to acquire information when information acquisition is overt, and revenue equivalence fails. Compte and Jehiel (2007) show in an IPV setup that an ascending dynamic auction induces more overt information acquisition and higher revenues than a sealed-bid auction. Hernando-Veciana (2009) compares the incentives to overtly acquire information in the English auction and the SPA, when bidders can learn about a common component or about a private component. In his model, it is *exogenous* which component information acquisition is about, while in my model, I endogenize the decision of information selection between the two components.

My paper also relates to the literature on *information choice* in games with strategic complementarities or substitutes, such as Cournot competition, beauty contests and investment games. Hellwig and Veldkamp (2009) ask whether bidders want to coordinate on the same or on different information channels about the same one-dimensional state of the world in a beauty contest game. They show that the choice of information relates to the complementarity of actions in their model: if actions are strategic complements, agents want to know what others know. If actions are strategic substitutes, agents want different information channels.

In a beauty contest game in Myatt and Wallace (2012), agents to choose between multiple information channels about a common state variable. Agents choose how clearly (endogenous noise) to listen to which of many available signals, that vary in accuracy (exogenous noise).

Gendron-Saulnier and Gordon (2017) fix the informativeness of signals, similar to my approach. In their paper, agents have the choice between multiple information channels, that all have the same informativeness: they are all Blackwell sufficient for each other. Information channels vary in the level of dependence they induce between the signals of agents. Actions exhibit strategic complementarities, as in the framework of Hellwig and Veldkamp (2009) and Myatt and Wallace (2012).

There are two major differences between my model and the three papers Hellwig and Veldkamp (2009), Myatt and Wallace (2012) and Gendron-Saulnier and Gordon (2017):[13] bidding functions do not exhibit strategic complementarities in the auction formats in my model (see e.g. Athey, 2002) which leads to a fundamentally different strategic problem. Further, in the above models, all channels contain information about the same single-dimensional payoff-relevant random variable (the one-dimensional state of the world). In contrast, in my model bidders choose about which component of the multidimensional state of the world to learn. Learning about their private component leaves them with an independent signal realization, irrespective of the information acquired by their opponent.

[13]See also Yang (2015) for flexible information acquisition in investment games with strategic complementarities and Denti (2017) for an unrestricted information acquisition technology in potential games.

# 2 Model

## 2.1 Payoffs

There are two risk-neutral bidders, indexed by $i \in \{1, 2\}$ who compete for one indivisible object. The reservation value of the auctioneer and the outside options of the bidders are zero.

The valuation for the object of bidder $i$, denoted by $V_i \in \mathbb{R}^+$, depends on two attributes: a common value component $S$ distributed on $[0, 1]$, that is equal for all bidders, and a private value component $T_i$ distributed on $[0, 1]$, the idiosyncratic taste parameter of bidder $i$.

The the common value component and the private value components $\{S, T_1, T_2\}$ are drawn mutually independent and identically, each with distribution function $H(\cdot)$, which admits a density function $h(\cdot)$.[14] That is, for all $i \in \{1, 2\}$, $m \in [0, 1]$, it holds that $H(m) = \Pr(S \leq m) = \Pr(T_i \leq m)$. The prior expected value of the components coincide: $\mathbb{E}[S] = \mathbb{E}[T_i]$.

The utility function for each bidder $i$ is

$$V_i = S + T_i.$$

Note that the private component of the other agent $j \neq i$ has no impact on the valuation of bidder $i$. In Section 5.1, I extend the class of admissible utility functions.

Fix a total value realization $v_i$. Any $s_i \in [\max\{v_i - 1, 0\}, \min\{v_i, 1\}]$ and $T_i = v_i - s$ is a feasible[15] combination of the components for this particular $v_i$. As the joint events $(S = s, V_i = v_i)$ and $(S = s, T_i = v_i - s)$ are the same, the density function of the random variable $V_i$, the overall valuation of bidder $i$, is

$$h_V(v_i) \coloneqq \int_{\max\{v_i-1,0\}}^{\min\{v_i,1\}} h(s)h(v_i - s)ds.$$

## 2.2 Information Structure

Neither the auctioneer, nor the bidders know the realization of any of the value components. Instead, bidders engage in information gathering about their valuations. The information choice of bidder $i$ is one of *information selection*: about which component should he learn.

Bidders choose one experiment $X_i$ which can be one of two random variables: bidders can learn either a random variable $X_i^T$ that is informative about their private component $T_i$, or a random variable $X_i^S$ that is informative about the common component $S$. Each signal $X_i \in \{X_i^T, X_i^S\}$ is uninformative about the other attribute. Both signals $X_i^T$ and $X_i^S$ consist of the same compact support $[0, 1]$ and a marginal probability distribution, conditional on the realization of its attribute $\{S, T_i\}$. The marginal distribution of the random variable $X_i^T$ or $X_i^S$ of bidder $i$ has a cumulative distribution function $F^T(\cdot|r)$ or $F^S(\cdot|r)$ for $r \in [0, 1]$, conditional on the state $T_i = r$ or $S = r$.

[14] The assumption of full support and existence of a density function $h(\cdot)$ is for clarity of the presentation. Results hold if there are only two realizations in the support.

[15] The interval has to account for the fact that each component is distributed with support $[0, 1]$. For example, if $v_i = 1.3$, the common component needs to be at least $s_i = \max\{v_i - 1, 0\} = 0.3$ for value $v_i$ to realize.

**Assumption.** *For $K \in \{S, T\}$, for all $r \in [0,1]$, the distribution $F^K(x_i|r)$ admits a density $f^K(x_i|r)$, such that:*

*(A1)* $\forall x_i \in [0,1] : f^S(x_i|r) = f^T(x_i|r) =: f(x_i|r)$.

*(A2)* $\forall x_i' > x_i$, $\frac{f^K(x_i'|r)}{f^K(x_i|r)}$ *strictly increasing in* $r$.

Assumption A1 implies that an experiment has the same conditional distribution function whether applied to $S$ or $T_i$. As all components are distributed identically, Assumption A1 implies the same informativeness on each available signal.[16] For clarity, I sometimes use the superscripts in the exposition to clarify about which component the signal is drawn.

The signals $X_i^S$ and $X_i^T$ satisfy a strong monotone likelihood ratio property (MLRP) in Assumption A2 which broadly speaking states that higher signal realizations are more indicative of higher states. Moreover, I assume that $f(\cdot|r)$ is continuously differentiable in $x_i$ for all $r$.

Bidders choose the probability $\rho_i$ of applying the signal on the common variable $S$. The information selection variable $\rho_i \in [0,1]$ is a mixed strategy:[17] With probability $\rho_i$, bidder $i$ performs an experiment about $S$. With probability $1 - \rho_i$, bidder $i$ learns about attribute $T_i$. Let $\boldsymbol{\rho} = \{\rho_1, \rho_2\}$ be the vector of information selection variables.

Due to the following assumptions, the private signals of bidders can only be interdependent via learning about the common variable $S$:

**Assumption** (CI)**.** $X_i^S \perp\!\!\!\perp X_j^S \mid S$.

**Assumption** (IN)**.** $X_i^T \perp\!\!\!\perp X_j^T$, *and* $X_i^T \perp\!\!\!\perp X_j^S$.

Assumption CI is a conditional independence assumption of $X_i^S$ and $X_j^S$ on $S$. Together with Assumption A2 (stating that $X_i^S$ and $S$ are affiliated) this implies that the random variables $X_1^S$ and $X_2^S$ are affiliated.[18] According to Assumption IN, if one bidder learns about his private component by observing $X_i^T$, his signal is independent from both signal $X_j^S$ and $X_j^T$ of his opponent $j$.

Let $F^S(x) := \Pr(X_i^S \leq x) = \int_0^1 F^S(x|s)h(s)ds$ be the unconditional distribution function of a bidders' private signal when he learns about component $S$, and let $f^S(x)$ be the corresponding density. Analogously, let $F^T(x) := \Pr(X_i^T \leq x) = \int_0^1 F^T(x|t)h(t)dt$ be the distribution when applying the experiment on $T_i$, and $f^T(x)$ the corresponding density. Note that $F^S(x) = F^T(x)$, due to the symmetry of signals and components.

After bidder $i$ chooses what to learn about, he observes signal $X_i$ with the following unconditional distribution function:

$$F(x) := \Pr(X_i \leq x|\rho_i) = (1 - \rho_i)F^T(x) + \rho_i F^S(x).$$

The unconditional distribution $F(x)$ is not a function of $\rho_i$, as applying the signal to both components results in the same distribution of signals due to $F^S(x) = F^T(x)$.

[16] I abstract away from bidders choosing to learn about a component only because it provides more information. Instead, the focus of this paper is to find what dependence bidders prefer between their signal given the same informativeness.

[17] A bidder always observes which experiment was performed for any randomization.

[18] For a formal definition of affiliation, see Appendix A.1, Definition 5. The affiliation between $X_i^S$ and $X_j^S$ follows from combining CI with MLRP.

**Observation 1.** *The unconditional distribution function of a signal about each component coincides for any information selection variable* $\rho_i$*:* $\forall x \in [0,1]$, $F^T(x) = F^S(x) = F(x)$.

The next binary example is useful to provide intuition in the following analysis.

**Example 1.** *Let* $s \in \{0,1\}$ *and* $t_i \in \{0,1\}$*, each with equal probability* $\frac{1}{2}$*. Thus,* $\Pr(v_i = 0) = \Pr(v_i = 2) = \frac{1}{4}$*, and* $\Pr(v_i = 1) = \frac{1}{2}$.

*For* $K \in \{S, T\}$ *and* $x_i \in [0,1]$*, the signal* $X_i^K$ *has conditional density* $f^K(x_i|0) = 2 - 2x_i$ *and* $f^K(x_i|1) = 2x_i$*. The unconditional signal distribution is* $F(x_i) = x_i$ *and the density is* $f(x_i) = 1$.

There are no costs associated with the information selection stage beyond the opportunity costs of not learning about the other value component. The *timing* is as follows.

1. An auction format is announced.
2. Nature draws $S, T_1, T_2$.
3. Bidders simultaneously and privately select their information $\boldsymbol{\rho} := \{\rho_1, \rho_2\}$.
4. Bidders privately observe their signal $X_i^S$ or $X_i^T$.
5. The auction takes place.

Information selection is *covert*: bidders do not observe which channel others chose to learn about, but make inference about it in equilibrium. Moreover, bidders select their information *after* the auction format is announced. This enables an analysis of the incentives of various auctions on information selection.

# 3 The Impact of Information Selection

## 3.1 Endogenous Correlation

With probability $(1 - \rho_1\rho_2)$ at least one bidder observes a signal about his private attribute $T_i$ and signals are independent by Assumption IN. With the remaining probability $\rho_1\rho_2$, bidders observe correlated signals about the same realization of the common attribute $S$. In this case, private signals $X_i^S$ and $X_j^S$ are independent conditional on the common value realization $s$ by Assumption CI.

Bidder $i$ forms a belief about the distribution of his opponent's signal, based on the source of his own signal, $X_i^T$ or $X_i^S$, and its realization $x_i \in [0,1]$. Bidder $i$ does not know whether his opponent $j$ observed a signal about $S$ or $T_j$, but draws inference if he expects his opponent to set $\rho_j > 0$, as the following cumulative distributions show.

Let $G^T(x_j|x_i, \rho_j) := \Pr(X_j \leq x_j | X_i^T = x_i, \rho_j)$ be the conditional cumulative distribution function of the 'source-free' signal realization $X_j$, from the perspective of bidder $i$ with a signal realization $X_i^T = x_i$. The distribution function $G^T$ does not depend on $\rho_i$, as it already conditions on bidder $i$ having observed a signal $X_i^T$ about $T_i$. If a bidder learns $X_i^T$, his signals contains no information about the other bidder due to Assumption IN. Therefore, using Observation 1, for all $x_i \in [0,1]$ and any information selection $\rho_j \in [0,1]$ of the opponent,

$$G^T(x_j|x_i,\rho_j) = F(x_j).$$

If bidder $i$ learns about his common component via $X_i^S$, his signal realization might bear information about his opponent's signal realization. Let $G^S(x_j|x_i,\rho_j) := \Pr(X_j \leq x_j|X_i^S = x_i, \rho_j)$ be the distribution function of the signal realization of bidder $j \neq i$, conditional on $X_i^S = x_i$.

$$G^S(x_j|x_i,\rho_j) = (1-\rho_j)F(x_j) + \rho_j \int_0^1 \frac{f^S(x_i|s)F^S(x_j|s)}{f(x_i)} h(s)ds.$$

The second summand accounts for the correlation in private information if the opponent $j$ also learns about the common component (with probability $\rho_j$). Then, signals are independent conditional on $S$ by Assumption CI.

## 3.2 Endogenous Value Setting

The degree of the winner's curse is endogenous in my model. If the opponent of bidder $i$ only learns about his private component $T_j$, his information is irrelevant for bidder $i$. Winning at any bid does not provide any further information for bidder $i$ beyond his private signal realization and there is no winner's curse.

If the other bidder $j$ learns about the common component, the event of winning contains information about $S$ for bidder $i$. If every bidder follows a symmetric and strictly increasing bidding function, winning indicates that bidder $j$ has a lower signal about $S$ than bidder $i$. This is bad news for the value of the object, and bidders shade their bid down to account for the effect of the winner's curse, to not overbid in case of a win.

Let bidder $i$ observe a signal $X_i^K$ for $K \in \{S,T\}$. His expected value of the object to bidder $i$, updated only based on his own signal realization is

$$\mathbb{E}\left[V_i|X_i^K = x_i\right] = \int_{\mathcal{V}} v_i h^K(v_i|x_i)dv_i,$$

where $h^K(v_i|x_i)$ is the following probability density function of the value $V_i$ for bidder $i$ conditional on his signal realization $X_i^K = x_i$ about component $K \in \{S, T_i\}$:

$$h^K(v_i|x_i) = \begin{cases} \frac{1}{f^S(x_i)} \int_0^1 \underbrace{f^S(x_i|s)h(s)h(v_i-s)}_{\substack{\text{joint event} \\ X_i^S=x_i, V_i=v_i, S=s}} ds \text{ if } K = S, \\ \frac{1}{f^T(x_i)} \int_0^1 \underbrace{f^T(x_i|t)h(t)h(v_i-t)}_{\substack{\text{joint event} \\ X_i^T=x_i, V_i=v_i, T_i=t}} dt \text{ if } K = T. \end{cases} \tag{1}$$

The following observation shows that any information selection leads to the same expected value of the object, conditional on that signal realization alone. This follows immediately from the symmetry of the distributions of the value components $S$ and $T_i$, $H(m) = \Pr(T_i \leq m) = \Pr(S \leq m)$ and the signals having the same density $f^T(x_i|r) = f^S(x_i|r)$ for each realization $x_i \in [0,1]$. That is, $h^S(v_i|x_i) = h^T(v_i|x_i)$.

**Observation 2.** *The object's expected value conditional on signal realization $x_i$ coincides for both signals $X_i^S$ and $X_i^T$: $\forall x_i \in [0,1]$, $\mathbb{E}\left[V_i|X_i^S = x_i\right] = \mathbb{E}\left[V_i|X_i^T = x_i\right]$.*

Both available signals $X_i^S$ and $X_i^T$ have equal informativeness about $V_i$, as they lead to the same posterior distribution over the total value. In equilibrium, bidders update about the value of the object, conditional on their signal, and conditional on the event of winning. Being pivotal bears information about the signal realization of the other bidder. The following expression is the value of bidder $i$ after observing an experiment about component $K \in \{S, T\}$, when the signal realization of the opponent is $X_j = x_j$, and the opponent selects $\rho_j$. For $K \in \{S, T_i\}$,

$$v^K(x_i, x_j|\rho_j) \coloneqq \mathbb{E}\left[V_i|X_i^K = x_i, X_j = x_j, \rho_j\right].$$

The above definition is based on a source-free signal realization $X_j = x_j$ of the other bidder, as bidder $i$ cannot observe whether it contains information about the common valuation $S$ or the private component $T_j$ of his opponent. However, it conditions on the information selection strategy of the other bidder, $\rho_j$. This is to capture that information selection is covert. While the choice of $\rho_j$ is unobservable to bidder $i$, he draws correct inference about it in equilibrium.

The following two value settings are nested in my model:

1. **Independent private values (IPV).** If $\rho_1 = \rho_2 = 0$, private signals $X_1^T$ and $X_2^T$ are independent. The expected value of bidder $i$ does not depend on the signal of bidder $j$:

$$v^T(x_i, x_j|\rho_j = 0) = \mathbb{E}\left[V_i|X_i^T = x_i\right] = \mathbb{E}\left[T_i|X_i^T = x_i\right] + \mathbb{E}\left[S\right].$$

2. **Common values/ mineral rights model (CV).** If $\rho_1 = \rho_2 = 1$, expected utility of the bidders is symmetric in the two private signals $X_1^S$ and $X_2^S$:

$$v^S(x_i, x_j|\rho_j = 1) = v^S(x_j, x_i|\rho_j = 1) = \mathbb{E}\left[T_i\right] + \mathbb{E}\left[S|X_i^S = x_i, X_j^S = x_j\right].$$

For example, fix the information choice of bidder $j$ at $\rho_j = 1$ such that he always learns his signal $X_j^S$. If bidder $i$ learns signal $X_i^S = x_i$ about the common component, his expected value is as described in above CV setting. If bidder $i$ instead learns about his private component via observing $X_i^T$, his estimate of the object when his opponent has signal realization $X_j^S = x_j$ is

$$v^T(x_i, x_j|\rho_j = 1) = \mathbb{E}\left[T_i|X_i^T = x_i\right] + \mathbb{E}\left[S|X_j^S = x_j\right].$$

Let bidder $j$ select $\rho_j = 1$ and learn about the common component via $X_j^S$ and consider Example 1. Figure 1 depicts the expected value of the object for bidder $i$, when he expects his opponent to have the same signal realization as himself, $X_j^S = x$. The blue dashed line is the expected value

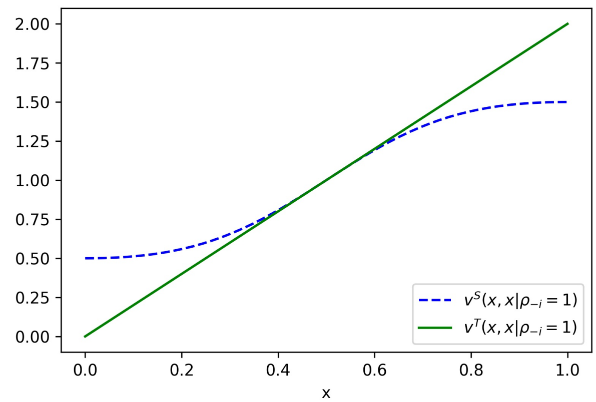


Figure 1: Expected valuation of bidder $i$ in Example 1, if he chooses to learn about component $K \in \{S, T_i\}$, the opponent learn about $S$, and both bidders have the same signal realization $x_i$.

$v_i^S(x_i, x_i|\rho_j = 1)$ for bidder $i$ in the CV framework with signal $X_i^S = x_i$. The green solid line is bidder $i$'s expected value $v^T(x, x|\rho_j = 1)$ if he learns about his private component. Expected value is increasing in the signal realization.[19] The function $v^S(x, x|\rho_j = 1)$ reacts slower to a change in the signal $x$ than $v^T(x, x|\rho_j = 1)$. This is because if a bidder learns about his private component, there is no dependence with his opponent, and therefore, no redundancy in private information. Receiving a low signal is worse news (and receiving a high signal is better news), if it contains information about the private component.

# 4 Second Price Auction

In this section, two bidders are competing for one indivisible object in a SPA, with no reserve price and an equal tie-breaking rule.[20] If the random vector $\boldsymbol{\rho}$ is exogenous and common knowledge, that is, when there is no information selection stage, the model reduces to Milgrom and Weber (1982). Under endogenous and covert information selection, bidders optimize their own information choice and make inference about the information source of their opponent in equilibrium, as it has an effect on the winning probability, the expected payment and the value of the object conditional on winning.

I consider the following class of equilibria:

**Definition 1** (Symmetric Bayes Nash equilibrium)
*In a symmetric Bayes Nash equilibrium, bidders*

- *select the same* $\rho_i = \rho^*$,

[19] The expected value $v^K(x_i, x_j|\rho_j = 1)$ of bidder $i$ with own signal $x_i$ and given the signal realization of the opponent $x_j$ is non-decreasing in both arguments. This follows from affiliation of $X_i^K$ with $X_j^S$ (Milgrom and Weber, 1982).

[20] For $N = 2$ bidders, the sealed bid SPA and the open English auction are strategically equivalent (see Milgrom and Weber, 1982). Furthermore, due to the assumption of strictly increasing bidding functions and no atoms in signal distributions, the probability of a tie is zero.

- *after observing $X_i^S = x$, bid $\beta^S(x)$,*
- *after observing $X_i^T = x$, bid $\beta^T(x)$,*

*where bidding functions $\beta^S(x)$ and $\beta^T(x)$ are pure and strictly increasing in $x$, and together with $\rho^*$ constitute mutually best responses.*

In the remainder of the paper, the term "equilibrium" refers to an object that satisfies the above definition. Let $CE \coloneqq \{\rho^*, \beta^S, \beta^T\}$ be a *candidate equilibrium*. The expected utility of bidder $i$ from learning about component $K \in \{S, T_i\}$ and bidding with $\beta_i$, who is facing an opponent who plays $CE$, is denoted by $EU(K, \beta_i|CE)$. It can be separated into his the expected gain $EG(K, \beta_i|CE)$ minus his expected payment $EP(K, \beta_i|CE)$:

$$EU(K, \beta_i|CE) \coloneqq EG(K, \beta_i|CE) - EP(K, \beta_i|CE). \tag{2}$$

First, consider the expected gain of bidder $i$. In the candidate equilibrium, bidder $i$ expects his opponent to learn $X_j^T$ and bid according to $\beta^T$ with probability $(1 - \rho^*)$; in this case, the expected gain of bidder $i$ is in line 3. With the remaining probability $\rho^*$, his opponent learns a signal $X_j^S$, bids according to $beta^S$. In this case, the expected gain of bidder $i$ is depicted in line 4.

$$EG(K, \beta_i|CE) \coloneqq (1 - \rho^*) \underbrace{\int_{\mathcal{V}} v_i \Pr(i \text{ wins}|v_i, X_i^K, \beta_i, X_j^T, \beta_j^T) h_{\mathcal{V}}(v_i) dv_i}_{\text{Expected gain of bidder i when } j \text{ learns } X_j^T} \tag{3}$$

$$+ \rho^* \underbrace{\int_{\mathcal{V}} v_i \Pr(i \text{ wins}|v_i, X_i^K, \beta_i, X_j^S, \beta_j^S) h_{\mathcal{V}}(v_i) dv_i}_{\text{Expected gain of bidder i when } j \text{ learns } X_j^S}. \tag{4}$$

Second, consider the expected payment of bidder $i$. In the SPA, if bidder $i$ wins he pays the bid of his opponent $j$. Consider the distribution of the signal of the opponent $j$, conditional on bidder $i$ having a higher signal. This distribution depends on both the information choices of the bidders and their bidding functions.

Let $L \in \{S, T_j\}$ be the component about which bidder $j$ learns signal $X_j^L$ and bids according to $\beta_j^L$. Whenever it is well-defined[21], define the cumulative distribution of bidder $j$'s signal realization *conditional* on bidder $i$ winning (when learning $X_i^K$ and bidding $\beta_i^K$):

$$H^K(x_j|\beta_i, \beta_j^L, X_j^L) \coloneqq \Pr(X_j^L \leq x_j|\beta_i(X_i^K) \geq \beta_j^L(X_j^L)). \tag{5}$$

Let $h^K(x_j|\beta_i, \beta_j^L, X_j^L)$ be the corresponding density, if it exists. With this information choice $K$ and $L$, and bidding functions $\beta_i, \beta_j^L$, the overall expected payment of bidder $i$ is:

$$EP(X_i^K, \beta_i|X_j^L, \beta_j^L) \coloneqq \Pr(\beta_i(X_i^K) \geq \beta_j^L(X_j^L)) \underbrace{\int_0^1 \beta_j^L(x_j) dH^K(x_j|\beta_i, \beta_j^L, X_j^L)}_{\text{payment conditional on winning}}. \tag{6}$$

The first factor is the overall probability of bidder $i$ winning. The second factor is the expected

[21]That is, if the probability of bidder $i$ winning is non-zero with $\beta_i(X_i^K = 1) > \beta^L(X_j^L = 0)$.

bid of $j$ that bidder $i$ has to pay conditional on winning.

Bidder $i$ does not observe which signal his opponent learns, but expects him to select $\rho^*$ in the candidate equilibrium. Based on this inference, bidder $i$'s expected payment with information choice $K$ and bidding function $\beta_i$ is

$$EP(X_i^K, \beta_i | CE) = (1 - \rho_j) EP(X_i^K, \beta_i | X_j^T, \beta_j^T) + \rho_j EP(X_i^K, \beta_i | X_j^S, \beta_j^S). \tag{7}$$

The first summand accounts for the possibility of the opponent having observed signal $X_j^T$ times the expected payment in this case. The second summand is the expected payment when facing an opponent with signal $X_j^S$, weighted with the probability $\rho_j$ of the occurrence of this event.

## 4.1 Information Selection in Equilibrium

The next theorem establishes the main result for the SPA. It shows that there is no learning about the common component in any equilibrium.

**Theorem 1.** *Information selection is unique in equilibrium,* $\rho^* = 0$.
*There exists an equilibrium in which* $\beta^T(x) = \mathbb{E}\left[V_i | X_i^T = x\right]$.

All proofs are in the appendix, unless stated otherwise. In the remainder of the section, I derive auxiliary results necessary to prove the above theorem.

First, consider any candidate equilibrium in which $\rho^* > 0$. Our goal is to establish that there exists a profitable deviation, as soon as there is positive dependence via learning about the common component. In general, a brute-force maximization approach to find the *best response* to a candidate equilibrium is a fruitless undertaking. This is because simultaneosly varying the information source and bidding function has adverse implications on the winning probability, expected payment and the posterior value of the object conditional on a win, and the overall effect on the payoff becomes intractable. Unless bidders follow the same bidding functions that allow some form of comparability, there is little that can be said about which strategy leads to a higher overall utility.

The trick is to *isolate* the effect on expected gain from the effect on expected payment conditional on a win. I establish existence of a deviation strategy that switches off any change in the expected gain *and* the winning probability. That is, by playing such a deviation strategy a bidder can guarantee himself the same expected gain and the same total winning probability as in in the candidate equilibrium. By picking the deviation strategy accordingly, we can concentrate on the effect on expected payment conditional on a win, as the the other components in Equation 2 are held constant. Critically hereby is to employ deviations that involve the same bidding functions between bidders even *after* the a deviation to a different information channel. This ensures that a bidder wins if and only if he has a higher signal than his opponent in certain cases. The following deviation strategy is strictly profitable whenever the candidate equilibrium contains $\rho^* > 0$.

**Definition 1.** *The deviation strategy (**DS**) for bidder i is the following strategy:*

- *deviate to* $\rho_i = 0$,
- *bid according to* $\beta^S(x_i)$ *for* $X_i^T = x_i$.

This deviation strategy takes the signal $X_i^T$ about the private component and maps it into a bid with bidding function $\beta^S$ as if it were the signal about the common component in the candidate equilibrium. While this is not necessarily the optimal bidding behavior learning $X_i^T$, it is strong enough to establish a profitable deviation over the combination $(X_i^S, \beta^S)$, which is part of any candidate equilibrium with $\rho^* > 0$.

## 4.2 Expected Gain

In this section, I compare the expected gain for bidder $i$ from DS to his expected gain from the CE with combination $(X_i^S, \beta^S)$, if he expects his opponent to play according to CE. I show that the expected probability of winning *conditional on a value realization $v_i$ for bidder $i$*, is identical in DS and in CE.

Fix a value $v_i$ for bidder $i$. There are two possibilities that can arise, depending on which information channel bidder $j$ chooses. Bidder $i$ does not know in which possibility he is in, as information selection is covert.

**Opponent with signal $\mathbf{X_j^T}$.** With probability $(1-\rho^*)$, the opponent of bidder $i$ learns signal $X_j^T$ about his private component, and follows the bidding function $\beta^T$. In this situation, a higher signal realization of bidder $i$ does not necessarily imply winning, as this depends on the interplay of the the bidding functions $\beta^S$ and $\beta^T$.

$$DS: \qquad \Pr(i \text{ wins}|v_i, \underbrace{X_i^T, \beta^S}_{DS}, X_j^T, \beta^T) = \Pr(\beta^S(X_i^T) \geq \beta^T(X_j^T)|v_i). \tag{8}$$

$$CE: \qquad \Pr(i \text{ wins}|v_i, \underbrace{X_i^S, \beta^S}_{CE}, X_j^T, \beta^T) = \Pr(\beta^S(X_i^S) \geq \beta^T(X_j^T)|v_i). \tag{9}$$

Neither playing $(X_i^S, \beta^S)$ in CE nor DS of bidder $i$ lead to correlation in private information, as by Assumption IN $X_j^T$ is independent from any signal of bidder $i$. Hence, the probability of a win conditional on any value $v_i$ is the same in Equation 8 and Equation 9, as the following lemma shows.

**Lemma 1.** *For all $v_i$,* $\Pr(i \textit{ wins}|v_i, \underbrace{X_i^S, \beta^S}_{CE}, X_j^T, \beta^T) = \Pr(i \textit{ wins}|v_i, \underbrace{X_i^T, \beta^S}_{DS}, X_j^T, \beta^T)$.

Note that Lemma 1 does not require bidder $i$ and $j$ to follow the same bidding function. The marginal distribution of both signals $X_i^S$ and $X_i^T$ of bidder $i$ coincide conditional on every value $v_i$. This follows as both signals have equal marginal distributions. As bidder $i$ follows the same bidding function in CE and DS, also the marginal distribution of *bids* coincides for each value $v_i$. As the signal of the opponent $X_j^T$ is independent from bidder $i$ for any information choice, the probability of winning is the same in CE and in DS.

**Opponent with signal $\mathbf{X_j^S}$.** With probability $\rho^*$, bidder $i$ faces an opponent who learns $X_j^S$ about his common component. In this case, both bidders follow the same bidding function $\beta^S$, and bidder $i$ wins if and only if his opponent has a lower signal than him.[22] The winning probabilities for bidder $i$

[22]Ties are ignored as they have zero probability.

conditional on $v_i$ in DS and in $(X_i^S, \beta^S)$ in CE are

$$DS: \qquad \Pr(i \text{ wins}|v_i, \underbrace{X_i^T, \beta^S}_{DS}, X_j^S, \beta^S) = \Pr(X_i^T \geq X_j^S|v_i). \tag{10}$$

$$CE: \qquad \Pr(i \text{ wins}|v_i, \underbrace{X_i^S, \beta^S}_{CE}, X_j^S, \beta^S) = \Pr(X_i^S \geq X_j^S|v_i). \tag{11}$$

For each total value realization $v_i$ for bidder $i$, the following theorem pins down the probability of having the highest signal in $(X_i^S, \beta^S)$ in CE and in DS.

**Proposition 1.** *For all total values $v_i$ for bidder $i$,*

$$\Pr(X_i^T \geq X_j^S|v_i) = \Pr(X_i^S \geq X_j^S|v_i) = \frac{1}{2}.$$

Hence, winning probability is equal for *every* value realization $v_i$ in both DS and CE:

$$\Pr(i \text{ wins}|v_i, \underbrace{X_i^S, \beta^S}_{CE}, X_j^S, \beta^S) = \Pr(i \text{ wins}|v_i, \underbrace{X_i^T, \beta^S}, X_j^S, \beta^S).$$

This proposition is more complicated to establish and does not follow from independence as does Lemma 1. This is because if the opponent learns $X_j^S$ about the common component, bidder $i$ has a choice between interdependence in signals (by choosing $X_i^S$ in CE) and independence (by choosing DS and $X_i^T$). Furthermore, the proposition crucially relies on the fact that there are only two bidders.[23]

It is instructive to consider how winning probability changes in different combinations of $S$ and $T_i$ for bidder $i$, when deviating to DS from CE. Proposition 1 establishes that winning probability conditional on any value realization $v_i$ is constant. Yet, the particular composition of states $S$ and $T_i$ of components, in which a bidder $i$ wins, changes.

Fix any total value realization $v_i$, and fix some feasible realization of the common component $s \in [\max\{0, v_i - 1\}, \min\{1, v_i\}]$. Then, $t_i = v_i - s$. If bidder $i$ plays according to CE with $(X_j^S, \beta^S)$ and faces an $X_j^S$-type opponent, his probability of winning at this combinations of $S$ and $T_i$ is

$$\Pr(X_i^S \geq X_j^S|S = s, T_i = v_i - s) = \int_0^1 f^S(x|s)F^S(x|s)dx = \frac{1}{2}.$$

If bidder $i$ plays DS instead, his winning probability for this combination $v_i$ and $s$ is

$$\Pr(X_i^T \geq X_j^S|S = s, T_i = v_i - s) = \int_0^1 f^T(x|v_i - s)F^S(x|s)dx.$$

The following lemma shows how DS shifts bidder $i$'s winning probability into states with a higher private component realization.

**Lemma 2.** *Fix $v_i \in (0, 2)$ and let bidder $j$ learn $X_j^S$ and bid according to $\beta^S$. With DS, bidder $i$ is strictly more (less) likely to win at $S < v_i/2$ ($S > v_i/2$) than with $(X_i^S, \beta^S)$ in CE. At $S = v_i/2$,*

[23] I extend the proposition in Section 5.2 to more than two bidders.

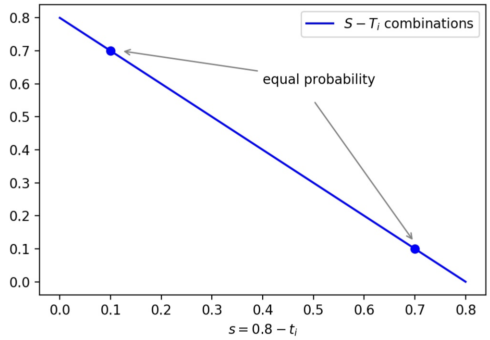


Figure 2: Iso-value curve for total values $v_i = 0.8$ of bidder $i$, showing different combination of feasible value components $S$ and $T_i$.

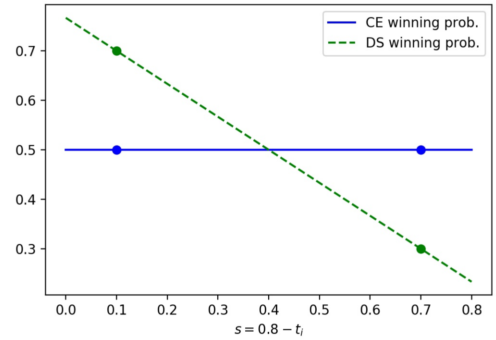


Figure 3: Winning probability in DS and $(X_i^S, \beta^S)$ in CE, for different $S$-$T_i$-combinations, given $v_i = 0.8$ and $X_j^S$. The two green dots in DS sum up 0.5.

*winning probability is equal in both strategies.*

By deviating to DS, for a given $v_i$, a bidder is strictly more likely to win in combinations that involve a high $T_i$, and strictly less likely to win in combination that involve a high $S$. For example, fix the value $v_i = 0.8$ for bidder $i$, that stems from any combination of the common component $S \in [0, 0.8]$ and private component $t_i = 0.8 - S$. This is depicted in Figure 2, where the diagonal line shows the iso-value curve, that consists of all feasible $S - T_i$ combinations that lead to the same overall value $v_i$ for bidder $i$.

In CE with $(X_i^S, \beta^S)$, bidder $i$ wins with equal probability of $\frac{1}{2}$ for *every* realization of $S \in [0.8]$ when facing an opponent $j$ who also learns about the common component. This is because both bidders have access to the same winning technology, and both learn about the same variable $S$.

With DS, bidder $i$ and bidder $j$ look at different value components. Due to the MLRP, higher signals are more likely for higher realizations of the value components. With DS, bidder $i$ is more likely to win in states with a high *private* component realization, and less likely to win with a high common component realization.

This is depicted in Figure 3. The x-axis shows the common component $S$ that is feasible with $v_i = 0.8$. For each feasible $s$ on the x-axis, there exists a unique realization of $t_i$ such that $v_i = 0.8$. The y-axis is the respective winning probability for such a realized pair $(S, T_i)$. The blue solid line at $y = \frac{1}{2}$ shows the winning probability with CE which is constant at one half. The green dashed line sketches the winning probability with DS. The two lines cross exactly at $v_i/2 = 0.4$.

In sum, the overall effect on the winning probability sums up to zero. Winning probability is the same in CE with $(X_i^S, \beta^S)$ and DS for every $v_i$. To provide intuition why the effect on overall winning probability given $v_i$ evaporates, fix the following two combinations: $(s = 0.1, t_i = 0.7)$ and $(s = 0.7, t_i = 0.1)$. Those two points are depicted by blue dots on the iso-value curve in Figure 2. As $S$ and $T_i$ are distributed identically and independently, both combinations have equal probability of $h(0.1)h(0.7)$.

In CE, winning probability of bidder $i$ is $\frac{1}{2}$ in both those possibilities. This is depicted by the two blue dots on the blue solid line at $y = 1/2$ in Figure 3. With DS, if $(s = 0.1, t_i = 0.7)$, the winning probability of bidder $i$ is no longer $\frac{1}{2}$, but higher due to the higher private component in comparison to the low common component. This is depicted by the green dot in the upper left corner of Figure 3. However, bidder $i$ loses the exact same winning probability in state $(s = 0.7, t_i = 0.1)$, as there it is his opponent who observes a signal about the higher state $s$, while bidder $i$ learns about the lower component realization $t_i$. This is depicted by the green dot in the lower right corner. The overall effect of the change in winning probability in the two combinations balances out to zero. In sum, overall probability of a win conditional on being in one of those two combinations, remains 1/2. This argument works for any two feasible symmetric combinations $(s = a, t_i = v_i - a)$ and $(s = v_i - a, t_i = a)$ for any $v_i$. Therefore, information selection shuffles the states in which bidder $i$ wins, while keeping the overall probability fixed.

To sum up, given any realization of the total value $v_i$, DS and $(X_i^S, \beta^S)$ in CE yield the same probability of winning if his opponent learns about the common component, snd if the opponent learns about his private component. The following corollary shows the impact of DS on the expected gain in Equation 3 and Equation 4 and on total winning probability in comparison to CE with $(X_i^S, \beta^S)$. It is an immediate implication of of Lemma 1 and Proposition 1, and the proof is therefore omitted.

**Corollary 1.** *Expected gain in CE with $(S, \beta^S)$ and DS coincide. The total winning probability is identical in DS and CE with $(S, \beta^S)$.*

As winning probability is the same for every $v_i$, it is also the same overall in CE and DS.

## 4.3 Expected Payment

The expected payment conditional on winning changes under the deviation strategy. In the following I show, that DS leads to a strictly lower payment by establishing a stochastic dominance order between the payment distributions with and without interdependence in private signals.

Consider the signal distribution of the opponent $j$, conditional on bidder $i$ winning in Equation 5. First, consider bidder $i$ facing a $X_j^T$-type opponent. If $\beta^S(x_i = 1) \leq \beta^T(x_i = 0)$, bidder $i$ has a zero-probability of winning in DS and in CE with $(X_i^S, \beta^S)$ against bidder $j$ bidding with $\beta^T$. Therefore, deviating to DS does not change the expected payment when facing a $X_j^T$-type opponent.

If $\beta^S(x_i = 1) > \beta^T(x_i = 0)$, bidder $i$ who employs bidding strategy $\beta^S$ has a non-zero winning probability when facing a $X_j^T$-type opponent. The distribution of signals of the loosing bidder $j$ is well-defined. It is $H^T(x_j|\beta^S, \beta^T, X_j^T)$ if bidder $i$ plays DS, and $H^S(x_j|\beta^S, \beta^T, X_j^T)$ if bidder $i$ plays CE with $(X_i^S, \beta^S)$.

If bidder $i$ faces a $X_j^S$-type opponent, the distribution of his opponent's signal is $H^T(x_j|\beta^S, \beta^S, X_j^S)$ if bidder $i$ plays DS, and $H^S(x_j|\beta^S, \beta^S, X_j^S)$ if bidder $i$ plays CE with $(S, \beta^S)$.

**Lemma 3.** *1. Opponent with signal $X_j^T$: for all $x_j \in [0, 1]$, if $\beta^S(1) > \beta^T(0)$, then*

$$H^S(x_j|\beta^S, \beta^T, X_j^T) = H^T(x_j|\beta^S, \beta^T, X_j^T);$$

*2. Opponent with signal $X_j^S$: $H^S(x_j|\beta^S, \beta^S, X_j^S)$ (strictly) first order stochastically dominates (FOSD) $H^T(x_j|\beta^S, \beta^S, X_j^S)$;*

*3. Overall expected payment is strictly lower under DS than in CE with $(X_i^S, \beta^S)$.*

The first property says that as long the opponent looks at his private component $X_j^T$, the expected distribution of payments of bidder $i$ in case of a win does not depend on bidder $i$'s information choice. Note that it does not rely on bidder $i$ and $j$ employing with the same bidding functions, but only bidder $i$ using the same $\beta^S$ for both his information channels $X_i^S$ and $X_j^S$. The property holds because if bidder $j$ learns $X_j^T$, his signal and thus his bid distribution is independent from both signals $X_i^S$ and $X_i^T$ of bidder $i$. Both these signals of bidder $i$ have the same marginal distribution via Observation 1. The argument is similar to the proof of Lemma 1 and relies on independence in private signals.

The second property establishes that if bidder $j$ learns $X_j^S$ about the common component, and both bidders follow the same bidding function $\beta^S$, the event of bidder $j$ having a signal below some $x_j$ *conditional on* bidder $i$ winning is more likely for every $x_j$. That is, the cumulative distribution of the second order statistic under interdependent signals with $X_i^S$ FOSD the distribution of the second order statistic under independence with $X_i^T$. By the FOSD, conditional on bidder $i$ winning, the signals and therefore the bids of the opponent are distributed lower in DS than in CE with $(X_i^S, \beta^S)$.

For a quick sketch[24] of the argument, the following expression is the signal distribution of $X_j^S$ of bidder $j$, conditional on bidder $i$ playing DS and winning (in this case signals of the two bidders are independent):

$$H^T(x_j|\beta^S, \beta^S, X_j^S) = 2\int_0^{x_j} f(\tilde{x}_j)\,(1 - F(\tilde{x}_j))\,d\tilde{x}_j = 2F(x_j) - F(x_j)^2. \tag{12}$$

This is the second order statistic of the two equally distributed independent signals $X_i^T$ and $X_j^S$, as bidder $i$ pays the second order statistic conditional on winning. Both bidders follow the same bidding function $\beta^S$, and bidder $i$ wins if and only if he has a higher signal than his opponent.

If bidder $i$ plays CE with $(X_i^S, \beta^S)$, conditional on bidder $i$ winning with $X_i^S$ and $\beta^S$, the distribution of his opponent's signal $X_j^S$ is the following expression:

$$H^S(x_j|\beta^S, \beta^S, X_j^S) = 2\int_0^{x_j}\int_0^1 f(\tilde{x}_j|s)\,(1 - F(\tilde{x}_j|s))\,h(s)ds d\tilde{x}_j = 2F(x_j) - \int_0^1 F(x_j|s)^2 h(s)ds. \tag{13}$$

This is the cumulative distribution function of the second order statistic under correlation between $X_i^S$ and $X_j^S$ via the common component $S$.

Comparing Equation 12 with Equation 13 shows that conditionally on a win, less correlation induces a lower distribution of the second order statistic and thus, a lower payment distribution. That is, for all $x_j \in (0,1)$, the Cauchy-Bunyakovsky-Schwarz (strong)[25] inequality establishes

$$F(x_j)^2 = \left(\int_0^1 F(x_j|s)h(s)ds\right)^2 < \underbrace{\int_0^1 h(s)ds}_{=1}\int_0^1 F(x_j|s)^2 h(s)ds.$$

[24]See the proof of Lemma 3 for a derivation of these cumulative distribution functions.

[25]For the strong Cauchy-Bunyakovsky-Schwarz inequality, see Footnote 33 in the proof of Lemma 3.

Hence, the probability of paying any bid $\beta^S(x_j)$ or below conditional on winning is lower when playing DS than when playing the candidate equilibrium with $X_j^S$. Conditional on winning, the lower the distribution of the opponent's signal (i.e. the lower the second order statistic), the lower the expected payment given a fixed bidding strategy $\beta^S$ of the opponent. Consider the limiting case of almost perfect correlation. Conditional on the event of winning, the bid of the other bidder is close to the own bid. Without correlation, the bid of the opponent conditional on a win is distributed independently. Conditional on winning, a bidder prefers his opponent to bid as low as possible. Positive interdependence raises the expected payment conditional on a win by increasing the distribution of the second order statistic in the sense of FOSD.

To sum up, when facing a $X_j^T$-type opponent, expected payment is the same in DS and CE with $(X_i^S, \beta^S)$. Conditional on a win against a $X_j^S$-type opponent, the payment distribution of bidder $i$ with DS is strictly dominated by the payment distribution with CE and $(X_i^S, \beta^S)$. Hence, the conditional payment is strictly lower in DS than in CE with $(X_i^S, \beta^S)$. As the bidding function $\beta^S$ is strictly increasing in the signal, this follows immediately via strong FOSD in Equation 6. By Corollary 1, the winning probability with DS is equal to the winning probability in CE. Hence, the unconditional expected payment is also strictly less with DS with $X_j^S$ of the opponent.

The probability to encounter a $X_j^S$-type opponent is non-zero in any candidate equilibrium with $\rho^* > 0$. Therefore, the third statement of Lemma 3 follows. Unconditional expected payment from DS is strictly less than in the candidate equilibrium with $(X_i^S, \beta^S)$.

## 4.4 Equilibrium and Social Surplus

The advantage of the deviation strategy DS is that it does not modify neither the overall probability of winning for each valuation $v_i$ nor the expected gain, but instead lowers the expected payment in case of a win due to less dependence between the signals of the two bidders. Combined, Corollary 1 and Lemma 3 establish that no $\rho^* > 0$ can be an equilibrium, as DS constitutes a strictly profitable deviation.

For Theorem 1 to hold we need to establish existence of an equilibrium with $\rho^* = 0$. In this case, both bidders learn about their private components, information is only relevant for the bidder who observes it, and bidders are in an IPV setup. Hence, due to Observation 2, a bidder is indifferent between the two signals. The value of information from both signals is the same, as both lead to no interdependence with the opponent, and both induce the same best response and posterior about the total value of the object.

**Social surplus** is maximized if a bidder with the highest expected private component $T_i$ receives the object. All bidders share the same common component $S$, which therefore plays no role for the social surplus. Ex-ante efficiency requires all bidders to learn only about their private component, to maximize the ex-ante expected social surplus. Information about the common component is not socially valuable, and available only by incurring the opportunity costs of not learning about the private component. Theorem 1 establishes that no equilibrium exists unless $\rho^* = 0$. The *SPA is ex-ante efficient* as it induces $\rho^* = 0$ and allocates efficiently.

# 5 Generalization

In this section, I analyze the incentives to select information in the SPA for a broader class of utility functions (Section 5.1) and discuss the applicability of my approach for the case of more than two bidders in Section 5.2. In the following, I restrict attention to pure information selection: bidders select information either about their private component via $\rho_i = 0$ or about their common component via $\rho_i = 1$. I show that my approach generalizes to a general class of utility functions, as long as they satisfy a marginal rate of substitution property.

## 5.1 General Utility Function

In the preceding parts of this paper, a bidder's overall utility function was symmetric, $V_i = S + T_i$. Next, consider a generalized class of utility functions $V_i = u(S, T_i)$ that satisfy the following properties:

1. $u(0, 0) \geq 0$, and $u(1, 1) < \infty$;
2. $u(\cdot, \cdot)$ is strictly increasing in both arguments;
3. for all $I \in [0, 2]$, $u(S, I - S)$ is non-increasing in $S \in [0, 1]$.

The first property binds the utility of a bidder above and below such that it is never strictly negative. The second property guarantees that any increase in either of his two components is strictly better for the bidder. The third property is a condition on the marginal rate of substitution between the two components $S \in [0, 1]$ and $T_i \in [0, 1]$, when their sum is constant at some $I \in [0, 2]$. The property states that by substituting $T_i$ with the same amount of $S$, the bidder is weakly worse off. If the utility function is differentiable in both arguments, the third property simply reduces to a marginal rate of substitution inequality: $\frac{\partial u(\cdot)}{\partial S} \leq \frac{\partial u(\cdot)}{\partial T_i}\Big|_{I=S+T_i}$.

A utility function, that satisfies above assumptions, is for example

$$V_i = \alpha S + (1 - \alpha) T_i$$

with $\alpha \in (0, \frac{1}{2}]$. For this particular example, it is straightforward to see that for any sum of the components $I = S + T_i$, we have $\frac{du(S, I-S)}{dS} \leq 0$ whenever $\alpha \leq \frac{1}{2}$. The following proposition extends the result for the SPA for this extended class of utility functions.

**Proposition 2.** *For all utility functions satisfying properties 1.-3., there exists no equilibrium of the SPA in which bidders learn about the common component via $\rho^* = 1$.*

The proof is by contradiction, along the lines of the technique developed in Section 4. For a sketch of the argument, consider $\rho^* = 1$ being a candidate equilibrium (CE). That is, in equilibrium both bidders learn only about their common component and expect their opponent to do the same. Then, bidder $i$ can play the following deviation strategy (DS) as in the preceding section and strictly increase his expected utility: *Set $\rho_i = 0$ and observe $X_i^T$, but bid according to bidding function $\beta^S$ that bidder $i$ uses in the candidate equilibrium with $X_i^S$.*

In contrast to the preceding section, the expected gain from DS will be different than in CE. By Proposition 1, with a symmetric utility function $V_i = S + T_i$, it holds for two bidders and any $v_i \in [0, 2]$

realization: $\Pr(X_i^T \geq X_j^S | v_i) = \Pr(X_i^S \geq X_j^S | v_i)$. The theorem conditions on all combinations of $S$ and $T_i$, that sum up to $I = v_i$, which corresponds to the same utility of $v_i$ for the symmetric utility function $V_i = S + T_i$. Hence, there is no need to differentiate between the sum of the two components and the overall utility for bidder $i$ from this component combination.

Note that for the general utility functions satisfying properties 1.-3., the statement of Proposition 1 holds exactly in the same manner when conditioning on $I = S + T_i$, but no longer on the value realization $v_i$, which might be different for the same sum $I$ of the two components.[26] That is, we have for all $I \in [0, 2]$

$$\Pr(X_i^T \geq X_j^S | I) = \Pr(X_i^S \geq X_j^S | I).$$

In DS and in CE, bidder $i$ follows $\beta^S$, the same bidding function as his opponent in the CE with $\rho^* = 1$. Hence, bidder $i$ wins whenever he has a higher signal realization than his opponent. Thus, the above is the probability of winning conditional on the sum $I$ of the two value components for bidder $i$.

This establishes that the bidder is equally likely to win, given the sum of the two components. In Section 4.2 I establish that while keeping the overall probability of a win for $v_i$ fixed, DS has an adverse effect on winning in different combinations of $S$ and $T_i$, as Lemma 2 depicts. Lemma 2 applies exactly in the same way for the case of fixing the sum of the two components $I$, instead of $v_i$. Replacing every $v_i$ in the proof by $I$ yields the result.

By deviating to DS, for a given $I$, a bidder is strictly more likely to win in combinations that involve a high $T_i$, and strictly less likely to win in combination that involve a high $S$. By property 3., a bidder prefers those combinations with a higher $T_i$ in which he wins more often over those with a low $T_i$ in which he loses winning probability. Hence, his expected gain from DS is weakly higher than from CE.

**Example 2.** *Let $V_i = 0.4S + 0.6T_i$ and consider the following density of signals for each component realization $r \in [0, 1]$: $f(x|r) = (2 - 2r) + (4r - 2)x$. For this linear example, the winning probability when deviating to DS is $\Pr(X_i^T \geq X_i^S | I, S = s) = \frac{I}{3} - \frac{2s}{3} + \frac{1}{2}$ for all $I \in [0, 2]$ and all feasible $s \in [\max\{0, I - 1\}, \min\{1, I\}]$.*

The winning probability at different realizations of $S$ for Example 2 is depicted in Figure 4. It shows how the winning probability varies in $s$ for a given sum of the components of bidder $i$, $I = 0.8$, if the bidder follows DS or plays the candidate equilibrium strategy $\rho^* = 1$ and $\beta^S$. For $I = 0.8$, the black dotted line is the winning probability of bidder $i$ with $s$ in the candidate equilibrium, and the blue solid line is his winning probability in $s$ from DS. Note that the $s$-axis ends at $s = I$: no higher $S$ is compatible with $I = 0.8$. It shows how winning probability under DS is reallocated from states with a high $S$ to states with a lower $S$ (into states that are more desirable for the bidder under property 3. of the utility function), while keeping the overall probability fixed.

The purple dotted line is the object's valuation of bidder $i$ for the specific $I - S$ combination, when his utility function is $V_i = 0.4S + 0.6T_i$ as in Example 2. It sketches that bidder $i$'s expected gain increases from playing DS due to a shift of winning probability from states with low $T_i$ to states with

[26] The steps of the proof for fixing $I$ instead of $v_i$ are exactly the same as for $v_i$, and the result follows by simply replacing every $v_i$ in the proof of Proposition 1 by $I$.

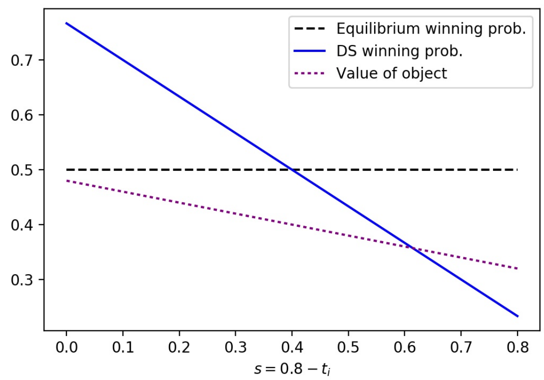


Figure 4: Winning probability in DS (blue solid line), the candidate equilibrium (black dashed line) and value of the object (purple dotted) with $\rho^* = 1$, $I = 0.8$, $f(x|r) = (2 - 2r) + (4r - 2)x$ and $V_i = 0.4S + 0.6T_i$.

high $T_i$ which the bidder values more.[27]

Note that Lemma 3 for the payment applies without changes: the expected payment is strictly lower under the DS, due to less interdependence between the bids in case of a win. This is because Lemma 3 does not rely on a specific functional form of the bidding function, but holds for any increasing symmetric function via FOSD.

Thus, a bidder is more likely to win when he values the object more. By property 3. of the utility function, he values an increase in $T_i$ more than in $S$. Instead of winning with probability $\frac{1}{2}$ at every realization of $S$, via DS a bidder shifts his winning probability to states with higher $T_i$ realization and lower $S$, and away from states with a high $S$ realization and lower $T_i$. As a result, the expected gain from DS is weakly better than in the CE with $\rho^* =$. A proof of this argument that accounts for the prior probability distribution over combinations of $S$ and $T_i$, is provided in the Appendix in the proof of Proposition 2.

This establishes, that the result that interdependence cannot be sustained in any equilibrium of the SPA is robust to a perturbation of the utility function into a direction, where the private component $T_i$ matters more for the bidder (property 3.).

A natural question is whether a perturbation into the other direction – making $S$ slightly more important for the bidder than $T_i$ – breaks the result. Consider the following utility function: $V_i = \left(\frac{1}{2} + \epsilon\right) S + \left(\frac{1}{2} - \epsilon\right) T_i$, with $\epsilon > 0$, such that $u(S, I - S)$ is strictly increasing in $S$. Can any equilibrium with learning about the common component via $\rho^* = 1$ be sustained under this utility function? For $\epsilon$ sufficiently small, the answer is No. Note that irrespective of the utility function, bidder $i$ can always guarantee himself a strictly lower payment by playing DS. His gain in payment from this deviation is bounded away from zero. Fix some $I = s + t_i$ for bidder $i$. Under DS, the bidder is more likely to win at states with high $T_i$ and low $S$, and less likely to win when $S$ is high (which he values more). In Figure 4, the purple dotted utility function would be increasing in $s$, showing that DS shifts his winning probability into unfavorable states combinations. Nevertheless, this loss in expected gain can be made arbitrarily close to zero by choosing $\epsilon$ sufficiently small. Therefore, the decrease in payment

[27] In the symmetric setup in Section 4 with the sum of the two components being the utility $V_i = S + T_i = I$, the purple dotted line is constant for every realization of $S$ for fixed $I$.

offsets the loss in gain for a sufficiently small $\epsilon$ in the utility function.

Hence, the argument that there cannot be learning about the common component in equilibrium is also robust to making the common component slightly more valuable to the bidder than the private component. Yet, increasing the marginal utility of $S$ further by increasing $\epsilon$ eventually breaks the predominance of the gain from lower payment over the lower expected gain from the object. Whether an equilibrium with learning about the common component can be sustained in equilibrium in such a case will depend on the primitives of the model: the utility function $V_i = u(S, T_i)$, the distributions of $S$ and $T_i$, and the signal distributions $f(x|r)$. The deviation strategy (DS) is no longer suitable for establishing non-existence in such a framework.

## 5.2 N Bidders

Consider a CE of the SPA with $\rho^* = 1$ and $\beta^S$. In the following I show the extension of Proposition 1 to the case of $N > 2$ bidders. Let the utility function be symmetric ($V_i = S + T_i$) as in Section 4.

Consider the same deviation strategy (DS) as for the case $N = 2$, in which bidder $i$ selects $\rho_i = 0$, observes $X_i^T$ and bids according to $\beta^S$ as if his signal were about the common component in CE.

In both strategies DS and CE, bidder $i$ wins if and only if he has a higher signal realization than all of his opponents, where ties can be ignored. Let $Y_i^S = \max_{j \neq i}\{X_1^S, ..., X_{i-1}^S, X_{i+1}^S, ..., X_N^S\}$ be the highest signal realization of all other bidders but bidder $i$ about the common component.

Due to independence conditional on $S$, the highest signal $Y_i^S$ of all other bidders has cumulative distribution function

$$G(y) = \int_0^1 F(y|s)^{N-1} h(s) ds.$$

For each total value realization $v_i$ for bidder $i$ the following theorem pins down the probability of winning under DS or CE, depending on whether he observes $X_i^T$ or $X_i^S$.

**Proposition 3.** *Let all $N - 1 \geq 2$ other bidders learn $X_{j \neq i}^S$ about the common component. Then, for all total values $v_i \in [0, 2]$ for bidder $i$:*

$$\Pr(X_i^T \geq Y_i^S | v_i) \geq \Pr(X_i^S \geq Y_i^S | v_i) = \frac{1}{N}.$$

*The inequality is strict for all $v_i \neq \{0, 1\}$.*

Let all other bidders learn about the common component $S$. Fix a total valuation for bidder $i$, by keeping the sum of the two components equal at $v_i = S + T_i$. The theorem says that, by selecting information about the private component $T_i$ instead of $S$, bidder $i$ can increase his probability of having the highest signal for all values $v_i$.

The difference of Proposition 3 to Proposition 1 with $N = 2$, in which winning probability is identical for all $v_i$, stems from difference in the first order statistic of a bidder's opponents. With $N = 2$, the distribution of the first order statistic of the other bidders is simply the signal distribution of a bidder's single opponent. Moreover, this distribution is the same as a bidder's own signal distribution. With more than one opponent, the first order statistic of the other bidders no longer coincides with the own signal distribution.

| | $(s = 0.7, t_i = 0.1)$ | $(s = 0.1, t_i = 0.7)$ | total winning prob. |
|---|---|---|---|
| CE | $1/N$ | $1/N$ | $1/N$ |
| DS | 0 | 1 | 1/2 |

Table 1: Probability of bidder $i$ winning in DS and CE with $\rho^* = 1$, conditional on $v_i = 1$. Both state combinations have equal probability of $h(0.1)h(0.7)$. Overall winning probability is higher with DS.

This becomes apparent in Figure 3 where different winning probabilities are described for the case of two bidders. Consider the same numerical example as before: $v_i = 0.8$ and either $(s = 0.1, t_i = 0.7)$, or $(s = 0.7, t_i = 0.1)$. With two bidders, bidder $i$ gains winning probability in $(s = 0.1, t_i = 0.7)$, but loses the same amount of winning probability in $(s = 0.7, t_i = 0.1)$, as his opponent is symmetric to him and the first order statistic is the same as his own signal distribution.

With more than two bidders, the gain of bidder $i$ from DS in state $(s = 0.1, t_i = 0.7)$ is larger than the winning probability that he loses in state $(s = 0.7, t_i = 0.1)$, when he bids against a higher first order statistic. The next example depicts this intuition for fully revealing signals.

**Example 3.** *Fix $v_i = 0.8$ and consider two $S$-$T_i$-combinations that are compatible with this total value realization for bidder $i$, $(s = 0.1, t_i = 0.7)$ and $(s = 0.7, t_i = 0.1)$. Both combinations occur with equal probability of $h(0.1)h(0.7)$ as $S$ and $T_i$ are drawn i.i.d.*

*Consider fully revealing signals about both value components $K \in \{S, T_i\}$, such that*

$$\Pr(X_i^K = x | K = r) = \begin{cases} 1 & \text{if} \quad x = r, \\ 0 & \text{otherwise.} \end{cases}$$

*If multiple bidders have the same highest signal realization, ties are broken evenly about who wins.*[28]

*If $(s = 0.7, t_i = 0.1)$, all $N-1$ other bidders learn a signal $X_j^S$ with realization $x_j = 0.7$. If bidder $i$ learns $X_i^S$ as well, he has signal realization $0.7$, and wins with probability $\frac{1}{N}$. If bidder $i$ observes signal $X_i^T$ instead about his private component, his signal realization is $0.1$ and he has zero probability of winning. These probabilities are summarized in the first column of Table 1.*

*If $(s = 0.1, t_i = 0.7)$, all other bidders observe a signal realization $x_j = 0.1$. If bidder $i$ learns about $S$, he also observes realization $0.1$ and wins with probability $\frac{1}{N}$. If bidder $i$ learns about his private component, his signal realization is $0.7$ and he wins with probability $1$. This is summarized in the second column of the Table 1.*

*Winning probability overall in DS is higher than in CE. In $(s = 0.1, t_i = 0.7)$, bidder $i$ has a lot of probability mass of winning to gain by learning about $T_i$. In state $(s = 0.7, t_i = 0.1)$, even if bidder $i$ learns about $S$, his probability of a win is not very high, since the first order statistic of the other bidders is elevated by the high realization of $S$. The gain in probability mass of winning in $(s = 0.1, t_i = 0.7)$ is larger than the loss in $(s = 0.7, t_i = 0.1)$.*

*This argument becomes apparent with $N \to \infty$. As the number of bidders increases and all other bidders learn about the common component, bidder $i$'s probability of winning with CE approaches zero in both $(s = 0.1, t_i = 0.7)$ and $(s = 0.7, t_i = 0.1)$. On the other hand, playing DS always guarantees*

[28] In the continuous version of my model, ties have zero probability. In this discrete example, ties occur with strictly positive probability, which requires a tie-breaking rule.

*bidder $i$ a win in state $(s = 0.1, t_i = 0.7)$. It is easy to see that when there are only two bidders, gain and loss in the two states are exactly equal: learning about either component yields the same overall probability $\frac{1}{2}$ of having the highest signal for bidder $i$ in above two state realizations. This is evident in the third column of Table 1 for $N = 2$.*

An immediate corollary of Proposition 3 is the following.

**Corollary 2.** *Let all the opponents of bidder $i$ learn about the common component by observing $X^S_{j \neq i}$. For $N > 2$, the overall probability of winning is strictly higher in DS than in CE.*

As Proposition 3 holds for each realization of $v_i$, it also holds overall and the proof is therefore omitted.

An overall higher probability of a win at every value realization $v_i$ might seem good news for the overall payoff in DS. Expected gain from DS is clearly strictly higher than the expected gain in CE. Complications arise in expected payment: total winning probability in DS is strictly higher than in CE. Hence, the expected payment conditional on a win is multiplied with a higher overall probability of winning in Equation 7. The separation approach in the expected utility − keep expected gain and total winning probability constant and focus on the expected payment − is no longer applicable as overall expected payment can strictly increase by switching from CE to DS and needs to be weighted against the gain in expected utility.

# 6 Alternative Auctions

In this section, I apply the developed technique to two further auction formats, the FPA (Subsection 6.1) and the all-pay auction (Subsection 6.2). As in the preceding section, I restrict attention to pure information selection, $\rho_i \in \{0, 1\}$. For the FPA, I show that $\rho^* = 1$ cannot be ruled out as an equilibrium with the developed approach. Furthermore, $\rho^* = 0$ is not robust in the FPA when introducing a small degree of correlation between the private component and the common component. In the all pay auction with more than two participants, bidders do not want to learn about the common component, and $\rho^* = 0$ is an equilibrium.

## 6.1 First Price Auction

Two bidders compete in a FPA with no reserve price.[29] Bidders can either learn about the common variable $S$ via observing the random variable $X^S_i$ or learn about the private variable $T_i$ via observing the random variable $X^T_i$, that is, $\rho_i \in \{0, 1\}$.

In section 4, I derived the necessary toolbox to show why $\rho^* = 1$ cannot arise in any equilibrium of the SPA: a bidder could play a certain deviation strategy that decreases correlation between his signal and the signal of the opponent. Then, bidding as if having observed $X^S_i$ but having truly observed $X^T_i$ yields bidder $i$ the same expected gain (Corollary 1) for a strictly lower payment (Proposition 3).

[29] As before in the SPA, ties have zero probability and can be ignored.

In the following I show why this argument *cannot* be used for the FPA to rule out $\rho^* = 1$. Let the candidate equilibrium be $\rho^* = 1$, and both bidders bid according to $\beta_f^S$. Consider the same deviation strategy as in the SPA for bidder $i$:

**Definition 2 ($\mathbf{DS^f}$).** *A deviation strategy ($DS^f$) for bidder $i$ in the FPA is the following strategy:*
- *deviate to $\rho_i = 0$ and observe $X_i^T$;*
- *bid according to $\beta_f^S(\cdot)$.*

The expected payoff from this deviation strategy is best evaluated by once again separating the expected gain from the expected payment. In the candidate equilibrium, bidder $i$ is sure that he faces a $X_j^S$-type opponent. Then, by Proposition 1 and Corollary 1, total winning probability is the same in DS and the candidate equilibrium. That is, $\Pr(X_i^S \geq X_j^S) = \Pr(X_i^T \geq X_j^S) = \frac{1}{2}$. Therefore, expected gain from $DS^f$ is the same as from the equilibrium bidding strategy. This immediately follows from Corollary 1, as the effect of DS on the expected gain coincides in the FPA and the SPA coincide. The difference between SPA and the FPA lies in their payment rule and not in the allocation decision.

Next, I show the effect of $DS^f$ on the expected payment. Similar to the SPA, define the cumulative signal distribution of bidder $i$, conditional on winning. The definition of this distribution captures his own information choice $K \in \{S, T_i\}$, the information choice of his opponent $L \in \{S, T_j\}$ and both bidding functions $\beta_i^K$ and $\beta_j^L$.

$$H_f^K(x_i|\beta_i^K, \beta_j^L, X_j^K) \coloneqq \Pr(X_i^K \leq x_i|\beta_i^K(X_i^K) \geq \beta_j^L(X_j^L)).$$

For $DS^f$, this distribution is the following. The joint event of the common component being $S = s$, bidder $i$ seeing $X_i^T = x_i$ and bidder $i$ winning with $\beta_f^S$ has density $h(s)f^T(x_i)F^S(x_i|s)$. As the distributions of both components are the same by Assumption A1, I drop the superscripts in the following. Integrating over all common states results yields the distribution:

$$\begin{aligned} H^T(x_i|\beta_f^S, \beta_f^S, X_j^S) =& \frac{\int_0^{x_i}\int_0^1 F(\tilde{x}|s)h(s)dsf(\tilde{x})d\tilde{x}}{\Pr(X_i^S \geq X_j^S)} \\ =& \frac{\int_0^{x_i} F(\tilde{x})f(\tilde{x})d\tilde{x}}{\frac{1}{2}} = F(x_i)^2. \end{aligned}$$

As $DS^f$ involves bidding with the same bidding function $\beta^S$ as the opponent, the above distribution of signals of bidder $i$ conditional on winning simplifies to the distribution of the first order statistics of two independent signals, $X_i^T$ and $X_j^S$, each drawn with identical distribution $F(\cdot)$.

Next, consider the cumulative distribution of signals of bidder $i$ who follows the candidate equilibrium strategy. The joint event $S = s$, $X_i^S = x_i$ and bidder $i$ winning with $\beta_f^S$ has density $h(s)f^S(x_i|s)F^S(x_i|s)$. This results in the following distribution, where I once again drop the superscripts.

$$
\begin{aligned}
H^S(x_i|\beta_f^S,\beta_f^S,X_j^S) =& \frac{\int_0^{x_i}\int_0^1 f(\tilde{x}|s)F(\tilde{x}|s)h(s)dsd\tilde{x}}{\Pr(X_i^S \geq X_j^S)} \\
=& \frac{\int_0^1\int_0^{x_i} f(\tilde{x}|s)F(\tilde{x}|s)d\tilde{x}h(s)ds}{\frac{1}{2}} \\
=& \int_0^1 F(x_i|s)^2 h(s)ds.
\end{aligned}
$$

Using the strict[30] Cauchy-Bunyakovski-Schwarz inequality we have for all $x \in (0,1)$,

$$
F(x_i)^2 = \left(\int_0^1 F(x_i|s)h(s)ds\right)^2 < \underbrace{\int_0^1 h(s)ds}_{=1.}\int_0^1 F(x_i|s)^2h(s)ds. \tag{14}
$$

The distribution of the first order statistic is strictly higher under interdependent signals, than under independent signals. This establishes FOSD, as $H^T(x_i|\beta_f^S,\beta_f^S,X_j^S) < H^S(x_i|\beta_f^S,\beta_f^S,X_j^S)$ for all $x \in (0,1)$. That is, $H^T(\cdot)$ is FOSD over $H^S(\cdot)$. This immediately translates into a strictly higher payment under $DS$ in case of a win as the bidding function $\beta_f^S$ is strictly increasing:

$$
\int_0^1 \beta_f^S(x_i)dH^S(x_i) < \int_0^1 \beta_f^S(x_i)dH^T(x_i).
$$

The expected payment conditional on a win is strictly lower under the original equilibrium strategy than under the constructed deviation $DS^f$. Not only does decreasing the correlation not help like in the SPA, but it hurts the agent. A bidder still wins with the same probability conditional on any value realization $v_i$ (this is an implication of Proposition 1 and the construction of $DS^f$ using the same bidding strategy as the equilibrium). However, by decreasing correlation with his opponent, a bidder is more likely to win at higher signal realizations which drives up his expected payment. This shows why $\rho^* = 1$ cannot be ruled out as an equilibrium by a deviation strategy of the same kind as in the SPA that decreases interdependence in private information.

**Correlation between the components.** As in the case with the SPA, an IPV equilibrium with $\rho^* = 0$ always exists. This is because if the opponent of bidder $i$ observes a signal about his *private* component, bidder $i$ is in an IPV setup. Then, bidder $i$ is indifferent between both information channels, as they both contain the same accuracy about the total value $v_i$ and each signal realization leads to the same best response due to Observation 2. Such an equilibrium is a 'trivial' equilibrium, as each bidder's information has neither an effect on interdependence between the signals, nor on total valuations.

Next, I analyze whether the trivial equilibrium with $\rho^* = 0$ is robust to a small degree of interdependence between the bidders. For this purpose, I introduce a slight perturbation into the informational structure. First, the common component $S$ realizes with distribution $H(\cdot)$, as in the Model Section 2. Then, the private components $T_1$ and $T_2$ are drawn. In contrast to the analysis before, with probability

[30] See footnote 33 for the strict inequality.

$\epsilon$ the common and private component of bidder $i$ are identical (which is unobserved): $T_i = S$. With probability $1 - \epsilon$, $T_i$ is drawn independently and identically with the same cumulative distribution $H(\cdot)$. Therefore, $\epsilon$ captures the correlation between each bidder's private component and the common component. In the analysis so far, $\epsilon = 0$. Furthermore, with $\epsilon > 0$ the IPV framework is ruled out as the signal of the opponent always contains relevant information about the common component irrespective of its source. That is, learning $X_i^T$ and $X_i^S$ contain information about both components.[31]

The next proposition shows that in the FPA, there cannot exist an equilibrium in which bidders learn signals $X_i^T$ about their private components.

**Proposition 4.** *For $\epsilon > 0$, there exists no symmetric equilibrium of the FPA with $\rho^* = 0$.*

The proof follows by combining the following two lemmas. Like in the SPA, the proof of the theorem is by contradiction. It relies on providing a deviation strategy and decomposing expected utility into an expected gain (which is the same in $\overline{DS}^f$ and the candidate equilibrium) and an expected payment (which is strictly less under $\overline{DS}^f$). I show that the following deviation strategy $\overline{DS}^f$ is a strictly profitable deviation.

**Definition 3.** *The deviation strategy ($\overline{DS}^f$) for bidder i in the FPA is the following strategy:*

- *deviate to $\rho_i = 1$,*
- *bid according to $\beta_f^T(x_i)$ for $X_i^S = x_i$.*

The deviation strategy $\overline{DS}^f$ involves changing the information selection strategy from learning $X_i^T$ to learning $X_i^S$, but not the bidding function. It requires a bidder to learn about the common component, but follow the same bidding function as if the bidder learned $X_i^T$. It is complementary to the deviation strategies considered before, as its purpose is to increase (not decrease) correlation while following the same bidding function.

The following lemma pins down the effect of $\overline{DS}^f$ on the winning probability for each object value and the expected gain from this deviation.

**Lemma 4.** *For each value $v_i$, the winning probability of bidder i in $\overline{DS}^f$ equals the winning probability in equilibrium under $\rho^* = 0$. The expected gain from $\overline{DS}^f$ equals the expected gain from the equilibrium with $\rho^*$.*

The proof uses parts of Proposition 1 for the special case of two bidders. As bidders follow the same bidding strategy $\beta_f^T$ in both the equilibrium and $\overline{DS}^f$, a winning bidder is a bidder with the highest signal realization. Therefore, the proof relies not necessarily on the optimal deviation strategy, but one that uses the same bidding function $\beta_f^T$ for tractability of the change in winning probability for each $v_i$. Overall expected gain from the candidate equilibrium and the deviation strategy $DS^f$ is the same.

The next lemma pins down the difference in expected payment between the equilibrium with $\rho^* = 0$ and under the deviation strategy $\overline{DS}^f$.

[31] An alternative perturbation is the following: both bidders make a small 'tremble' when choosing their information source. With probability $1 - \epsilon$ they observe a signal about their preferred value component; with probability $\epsilon$ they perform an experiment on the wrong component. This perturbation yields the same results on equilibrium existence as the one introduced in this section.

**Lemma 5.** *Let $\epsilon > 0$. The expected payment with $\overline{DS}^f$ in the FPA is strictly less than in the equilibrium with $\rho^* = 0$.*

The argument is similar to the one developed above to show that $\rho^* = 1$ cannot profit from $DS^f$. Intuitively, achieving a stronger dependence with the opponent reduces the 'money left on the table' in the FPA. A bidder pays his own bid. Conditional on the event of winning, he prefers to outbid his opponent by as little as possible. Whenever the perturbation is inactive as the opponent bidders observed his private component signal $X_i^T$ as selected by $\rho^* = 0$, there is no difference between behaving as in equilibrium and following $\overline{DS}^f$. The deviation strategy comes into play whenever the opponent trembled and observed $X_i^S$. In this case, a bidder is more likely to observe a signal about the common component and be more correlated with the opponent under the deviation strategy $\overline{DS}^f$ than under the equilibrium strategy. As in both cases, bidders follow the same bidding function, the FOSD argument holds. The expected payment that a bidder has to pay in case of a win is strictly higher with less interdependence, as a bidder is more likely to win when he places a low bid (with higher dependence his opponent is less likely to outbid him in this range). Similarly, with more dependence, a bidder is less likely to win when placing a high bid, as his opponent is more likely to outbid him in the range of higher bids. Expected payment is strictly higher under less correlation, while expected gain is constant. And under the deviation strategy the event of higher correlation (when both bidders observed a signal about $S$) is more likely to occur.

To sum up Lemma 4 and Lemma 5, the deviation strategy does not change the winning probability or the expected gain from participating, but strictly decreases expected payment. As increasing the dependence in private information with the opponent comes without a loss for expected value, due to the particular construction of $\overline{DS}^f$, it constitutes a strictly profitable deviation. Therefore, $\overline{DS}^f$ is a strictly profitable deviation. The equilibrium $\rho^* = 0$ is not robust to the perturbation of the information structure.

## 6.2 All-Pay Auction

Consider an all-pay first price auction with $N$ bidders. Bidders submit bids $b_i$ as a function of their signal realization $X_i^T$ or $X_i^S$. Payment and allocation rule result in the following payoff $W_i$ for bidder $i$ who places bid $b_i$:

$$W_i = \begin{cases} V_i - b_i \text{ if } b_i > \max_{j \neq i} b_j \\ \frac{V_i}{\#\{k: b_k = b_i\}} - b_i \text{ if } b_i = \max_{j \neq i} b_j \\ -b_i \text{ if } b_i < \max_{j \neq i} b_j \end{cases}$$

Bidders always pay their bid, irrespective of the event of winning. They win if they submitted a higher bid than their opponents. Krishna and Morgan (1997) analyze the all-pay auction in a symmetric interdependent value framework. They show when a symmetric equilibrium in increasing strategies exists.

Denote the bidding function in a candidate equilibrium of the all-pay auction after learning $X_i^S$ by $\beta_a^S$, and after learning $X_i^T$ by $\beta_a^T$. The next theorem and lemma establish the main result for the all-pay auction about information selection and existence in equilibrium.

**Proposition 5.** *For $N > 2$, there exists no equilibrium of the all-pay auction with $\rho^* = 1$.*

Learning about the common component cannot arise in equilibrium. Similar to the proof technique of the SPA, I establish the result by constructing a deviation strategy and decompose it in expected gain and expected payment. It allows an application of Proposition 3 and enables a tractable payoff comparison. By contradiction, consider a candidate equilibrium of the all-pay auction with $\rho^* = 1$ in which all participants bid according to some increasing function $\beta_a^S(x)$.

**Definition 4 ($\mathbf{DS^a}$).** *A deviation strategy ($DS^a$) for bidder $i$ in the all-pay auction is:*

- *deviate to $\rho_i = 0$ and observe $X_i^T$,*
- *bid according to $\beta_a^S(X_i^T)$.*

The deviation strategy $DS^a$ requires a bidder to change the source of his signal (from $X_i^S$ to $X_i^T$), but follow the same bidding function $\beta_a^S(\cdot)$ as before. Let the utility from the candidate equilibrium (CE) with $\rho^* = 1$ and $\beta_a^S$ be $EU(S, \beta_a^S|CE)$. Let the expected utility from $DS^a$ be $EU(T_i, \beta_a^S|CE)$. The proof of Proposition 5 shows that $EU(S, \beta_a^S|CE) < EU(T_i, \beta_a^S|CE)$ for $N > 2$.

In the all-pay auction, a bidder pays his own bid. In CE, the expected payment of bidder $i$ is

$$\int_0^1 \beta_a^S(x_i) f(x_i) dx_i.$$

In $DS_a^S$, expected payment is exactly the same, as both of bidder $i$'s available signals $X_i^T$ and $X_i^S$ induce the same marginal distribution $f(x_i)$. Hence, expected payment in CE and $DS^a$ is the same.

The expected gain is strictly higher in $DS^a$, as due to Proposition 3, the probability of a win is strictly larger at almost all total values $v_i$ if there are more than two bidders.

For the case of two bidders, the expected payment in $DS^a$ and CE with $\rho^* = 1$ is the same as a bidder's bid (and thus, payment) distribution is the same. In contrast to the $N > 2$ bidder case, expected gain is also the same in CE and $DS^a$. That is, the expected overall utility of bidder $i$ from CE and from $DS^a$ is identical. Whether there exists a strictly profitable deviation over a CE with $\rho^* = 1$ will depend on the characteristics of the signals. One might expect that generically, as $\beta^S$ is constructed as a best response in CE after seeing $X_i^S$, there is no reason why it should also constitute a best response after seeing $X_i^T$, and the bidder could strictly increase his payoff by playing a best response to $X_i^T$.

**Lemma 6.** *For $N \geq 2$, there exists an equilibrium with $\rho^* = 0$.*

The proof is by construction: learning only about the private component $T_i$ and bidding according to the usual IPV bidding function for the all-pay auction $\beta_a^T(x) = \int_0^x \mathbb{E}\left[V_i|X_i^T = \tilde{x}\right] f^T(\tilde{x}) d\tilde{x}$ constitutes a best response to this particular information choice when the opponents also select $\rho^* = 0$ and follow the same bidding function.

Moreover, if $\rho^* = 0$, no other bidder knows anything of relevance to other bidders. Signal realizations of other bidders are independent from one's own signal for any information selection. The value of information conditional on one's signal alone is equal no matter which component the signal was applied to. Due to Observation 2, any signal realization results in the same best response. Bith available signals have the same value of information for a bidder, if the opponents play CE. This establishes existence of an equilibrium with $\rho^* = 0$.

# 7 Conclusion

If bidders cannot consider all possible information, a question of *which* variables to learn about arises. I analyze this question in the context of auctions. In takeover auctions, out of all the multidimensional information available about the target, which characteristics do bidders choose to focus on? Do they want to know what matters to others – a common variable like the book value – which induces interdependence in private information? Or do bidders prefer to focus on a private component like their specific R&D synergies and receive independent private signals? Bidders are equally well-informed about the object's total value whether they select a signal about the common or the private component.

The focus of this paper is on information selection, specifically *which* payoff-relevant variable to learn about. This contrasts with the literature on information acquisition, which usually asks *how much* information about a single payoff relevant variable a bidder acquires.

In the SPA, information selection in equilibrium is unique. Bidders learn only about their private component. Any candidate equilibrium in which bidders learn with non-zero probability about the common component can be ruled out by an appropriate deviation strategy. The deviation strategy uses the same bidding functions as the candidate equilibrium but induces independent private signals by learning only about the private component. By employing such a deviation strategy, a bidder strictly decreases his expected payment but retains his overall gain and winning probability. By decreasing correlation via learning about the private component, a bidder is more likely to win in states with a high *private* component, and less likely to win in states with a high *common* component, while there is no effect on the overall winning probability.

This paper explores the impact of a selling mechanism on the type of information bidders select. Information about the common component simplifies coordination and is informative about other bidder's bids. However, learning about a common component that matters equally for all bidders is socially wasteful, as this information comes at the opportunity cost of not learning socially valuable information about the private components. A designer who wishes to maximize efficiency should take into consideration, that his auction choice might affect about which value components bidders learn. My analysis suggests that, in such a simplified setting, the SPA is a good choice, as it is ex-ante efficient. It induces learning only about the socially relevant variable and allocates the good efficiently. An IPV setup arises endogenously.

# A Appendix

## A.1 Affiliation and Accuracy

The following definition introduces the concept of affiliation between random variables. Affiliation is a strong form of positive correlation, and is a widely used model of statistical dependence in Economics at the latest since the contribution of Milgrom and Weber (1982).[32]

[32]The concept of affiliation is known in the statistical literature as a multivariate total positivity order $MTP_2$ (Karlin and Rinott, 1980). For a comparison of affiliation with other forms of positive correlation, see de Castro (2009) in a context of auctions, and Shaked and Shanthikumar (2007) for a general account of positive dependence orders.

**Definition 5** (Milgrom and Weber (1982))**.** *Consider real-valued random variables $Z_1, ..., Z_k$, and denote a vector of realizations by $\boldsymbol{z} := \{z_1, ..., z_k\}$. Let $f(\boldsymbol{z})$ be the density of the realization vector $\boldsymbol{z}$. Denote by $\boldsymbol{z} \vee \boldsymbol{z}'$ the component-wise maximum, and denote by $\boldsymbol{z} \wedge \boldsymbol{z}'$ the component-wise minimum of the two vectors $\boldsymbol{z}$ and $\boldsymbol{z}'$. Then, the random variables $Z_1, ..., Z_k$ are said to be affiliated if*

$$\text{for all } \boldsymbol{z}, \boldsymbol{z}' : \quad f(\boldsymbol{z} \vee \boldsymbol{z}')f(\boldsymbol{z} \wedge \boldsymbol{z}') \geq f(\boldsymbol{z})f(\boldsymbol{z}').$$

**Observation 3.** *$X_1^S$ and $X_2^S$ are affiliated.*

This follows from Milgrom and Weber (1982). By Theorem 1, part (ii) in their model, the random variables $X_1^S, X_2^S$ and $S$ are affiliated if their density can be expressed as the product of affiliated non-negative functions. We have $f(x_1, x_2, s) = f^S(x_1|s)f^S(x_2|s)h(s)$ with $f^S(\cdot)$ being non-negative and affiliated due to the strong MLRP. By Theorem 4 in Milgrom and Weber (1982), as the triple of variables $X_1^S, X_2^S, S$ are affiliated, so are the two variables $X_1^S$ and $X_2^S$.

Note that independence is a special case of affiliation, where above inequality in Definition 5 holds with equality for all realizations $\boldsymbol{z}$ and $\boldsymbol{z}'$. This implies that $X_i^T$ and $X_j^S$ are affiliated, and $X_i^T$ and $X_j^T$ are affiliated, as they are independent due to Assumption IN.

## A.2 Proofs

*(The proof of Theorem 1 follows after the proof of Lemma 3, by combining the auxiliary results in Lemma 1, Proposition 1 and Lemma 3.)*

***Proof of Lemma 1.*** The distribution of $X_i^S$ conditional on $v_i$ and $X_i^T$ conditional on $v_i$ coincide. This is because the density of realization $x_i$ conditional on $v_i$ is $\frac{h^S(v_i|x_i)f^S(x_i)}{h_{\mathcal{V}}(v_i)}$, where $h^S(v_i|x_i)$ as defined in Equation 1. As $h^S(v_i|x_i) = h^T(v_i|x_i)$, and $f^S(x_i) = f^T(x_i)$ via Observation 1, this establishes that the signals $X_i^S$ and $X_i^T$ of bidder $i$ are equally distributed conditional on $v_i$. Therefore, also the marginal distributions $\beta^S(X_i^S)$ and $\beta^S(X_i^T)$ coincide, conditional on $v_i$.

Due to Assumption IN, any signal of bidder $i$, $X_i^S$ and $X_i^T$, is independent from $X_j^T$. As functions of independent random variables are independent themselves, for any information choice $K \in \{S, T_i\}$ of bidder $i$, the random variable $\beta^S(X_i^K)$ is independent from $\beta^T(X_j^T)$. Therefore,

$$\Pr(\beta^S(X_i^T) \geq \beta^T(X_j^T)|v_i) = \Pr(\beta^S(X_i^S) \geq \beta^T(X_j^T)|v_i),$$

which establishes the proposition for the winning probabilities in Equation 8 for DS and Equation 9 for CE. □

***Proof of Proposition 1.*** For $v_i = 0$, we have $s = 0$ and $t_i = 0$. Any information selection leads to a density $f(x|0)$, as both signals $X_i^S$ and $X_i^T$ have same density. The density of an opponent with signal $X_j^S$ is $F(x|0)$. The probability of having the highest signal is $\int_0^1 f(x|0)F(x|0)dx_i = \frac{1}{2}$. Similarly, for the total value to be $v_i = 2$, the components need to be $s = 1$ and $t_i = 1$. Then, the probability of having the highest signal with any signal is $\int_0^1 f(x|1)F(x|1)dx = \frac{1}{2}$.

Fix a total value for bidder $i$ at $v_i \in (0, 2)$. Define the set the common component $S$, that is feasible under this $v_i$ realization as $\mathcal{S}(v_i) := \{s \in S : \exists t_i \in [0, 1] : v_i = s + t_i\} = [\max\{0, v_i - 1\}, \min\{1, v_i\}]$.

E.g., if $v_i \geq 1$, we have $\mathcal{S}(v_i) = [v_i - 1, 1]$. If $v_i < 1$, we have $\mathcal{S}(v_i) = [0, v_i]$. Define $\hat{s}(v_i)$ that bisects this interval: $\hat{s}(v_i) \coloneqq \frac{\max\{0, v_i - 1\} + \min\{1, v_i\}}{2} = \frac{v_i}{2}$.

Conditional on the value for bidder $i$ being $v_i$, the density of the common component being equal to $s$ is $h(s|v_i) \coloneqq \frac{h(s)h(v_i - s)}{h_{\mathcal{V}}(v_i)}$. This is due to the fact that $S$ and $T_i$ are drawn from an identical distribution with density $h(\cdot)$. Note that $\int_{\mathcal{S}(v_i)} h(s|v_i) = 1$, and $h(s|v_i) = h(v_i - s|v_i)$, as $h(s|v_i)$ is symmetric around $\hat{s}(v_i)$.

If bidder $i$ learns $X_i^S$ and his opponent learns $X_j^S$, the probability of winning is

$$\begin{aligned}
\Pr(X_i^S \geq X_j^S|v_i) &= \int_{\mathcal{S}(v_i)} \int_0^1 f(x|s)F(x|s)h(s|v_i)dxds \\
&= \int_{\mathcal{S}(v_i)} \underbrace{\left[\frac{1}{2}F(x|s)\right]_0^1}_{=1/2} h(s|v_i)ds \\
&= \frac{1}{2}\int_{\mathcal{S}(v_i)} h(s|v_i)ds = \frac{1}{2}.
\end{aligned}$$

If the common component is $s$, then, conditional on $v_i$, bidder $i$ observes a signal about his private component $t_i = v_i - s$. If bidder $i$ learns about his private components via observing $X_i^T$, his probability of a win is the following.

$$\begin{aligned}
\Pr(X_i^T \geq X_j^S|v_i) &= \int_{\mathcal{S}(v_i)} \int_0^1 f(x|v_i - s)F(x|s)h(s|v_i)dxds && (15)\\
&= \int_{\max\{v_i-1,0\}}^{\hat{s}(v_i)} \int_0^1 f(x|v_i - s)F(x|s)h(s|v_i)dxds && (16)\\
&\quad + \int_{\hat{s}(v_i)}^{\min\{v_i,1\}} \int_0^1 f(x|v_i - s)F(x|s)h(s|v_i)dxds && (17)
\end{aligned}$$

The last step followed by splitting up the integral in two intervals. Consider the second integral. Using relabeling and integration by parts, we have

$$\begin{aligned}
&\int_{\hat{s}(v_i)}^{\min\{v_i,1\}} \int_0^1 f(x|v_i - s)F(x|s)h(s|v_i)dxds \\
=& \int_{\max\{v_i-1,0\}}^{\hat{s}(v_i)} \int_0^1 f(x|s)F(x|v_i - s)h(v_i - s|v_i)dxds \\
=& \int_{\max\{v_i-1,0\}}^{\hat{s}(v_i)} \left( \underbrace{[F(x|s)F(x|v_i - s)]_0^1}_{=1} - \int_0^1 f(x|v_i - s)F(x|s)dx \right) h(v_i - s|v_i)ds \\
=& \int_{\max\{v_i-1,0\}}^{\hat{s}(v_i)} h(v_i - s|v_i)ds - \int_{\max\{v_i-1,0\}}^{\hat{s}(v_i)} \int_0^1 f(x|v_i - s)F(x|s)dxh(v_i - s|v_i)ds \\
=& \frac{1}{2} - \int_{\max\{v_i-1,0\}}^{\hat{s}(v_i)} \int_0^1 f(x|v_i - s)F(x|s)h(s|v_i)dxds
\end{aligned}$$

where the last step followed by $h(s|v_i) = h(v_i - s)$ and $\int\limits_{\mathcal{S}(v_i)} h(s|v_i)ds = 1$. Plugging this back into Equation 17 yields the result, $\Pr(X_i^T \geq X_j^S|v_i) = \frac{1}{2} = \Pr(X_i^S \geq X_j^S|v_i)$.

□

***Proof of Lemma 2.*** As $f^S(x|r) = f^T(x|r)$ and $F^S(x|r) = F^T(x|r)$, I drop the superscript. Fix the value for bidder $i$, $v_i \in (0, 2)$.

For every feasible $s$ that can arise with $v_i$, if both bidders learn about $s$ and bid with $\beta^S$, the winning probability is $\frac{1}{2}$:

$$\Pr(X_i^S \geq X_j^S|S = s, T_i = v_i - s) = \int_0^1 f(x|s)F(x|s)dx = \frac{1}{2}.$$

Now consider the winning probability of bidder $i$ with DS when facing opponent with signal $X_j^S$. For $s = \frac{v_i}{2} = I - s$, it is immediate that $\int_0^1 f(x|v_i - s)F(x|s)dx = \int_0^1 f(x|s)F(x|s)dx = \frac{1}{2}$.

Take any $s < \frac{v_i}{2}$. A consequence of the strong MLRP is FOSD. Thus, for every $x \in (0, 1)$: $F(x|s) > F(x|v_i - s)$ for $s < v_i - s$. Hence, winning probability in DS is

$$\int_0^1 f(x|v_i - s)F(x|s)dx > \int_0^1 f(x|v_i - s)F(x|v_i - s)dx = \frac{1}{2}.$$

Therefore, the winning probability is larger when learning $X_i^T$ about the private value component, if the private component realization $t_i = v_i - s$ is larger than the common component realization $s$.

Finally, take any $s > \frac{v_i}{2}$. Similarly, due to the strong MLRP we have $F(x|s) < F(x|v_i - s)$ for all $x_i \in (0, 1)$. Thus,

$$\int_0^1 f(x|v_i - s)F(x|s)dx < \int_0^1 f(x|v_i - s)F(x|v_i - s)dx = \frac{1}{2}.$$

□

***Proof of Lemma 3.*** Consider statement 1. of the Lemma. For all realizations $x_j \in [0, 1]$, we have:

$$H^K(x_j|\beta^S, \beta^T, X_j^T) = \frac{\Pr(X_j^T \leq x_j, \beta^S(X_i^K) \geq \beta^T(X_j^T))}{\Pr(\beta^S(X_i^K) \geq \beta^T(X_j^T))}. \tag{18}$$

The event that the opponent has signal realization $X_j^T = x_j$ and bidder $i$ has a higher signal when learning about $K \in \{S, T_i\}$ has density $\Pr(\beta_i^S(X_i^K) \geq \beta_j^T(x_j))f^T(x_j)$. Note that $\Pr(\beta^S(X_i^T) \geq \beta^T(x_j)) = \Pr(\beta^S(X_i^S) \geq \beta^T(x_j))$ due to Assumption IN and the same marginal distribution of $X_i^S$ and $X_i^T$ in Observation 1. Therefore, the numerator can be rewritten in the following way and does not depend on the information channel of bidder $i$:

$$\int_0^{x_j} \Pr(\beta^S(X_i^S) \geq \beta^T(\tilde{x}_j))f^T(\tilde{x}_j)d\tilde{x}_j = \int_0^{x_j} \Pr(\beta^S(X_i^T) \geq \beta^T(\tilde{x}_j))f^T(\tilde{x}_j)d\tilde{x}_j.$$

Next, I establish that the the denominator in Equation 18 is equal in CE and in DS. By Lemma

1, we have for all $v_i$, $\Pr(\beta^S(X_i^S) \geq \beta^T(X_j^T)|v_i) = \Pr(\beta^S(X_i^T) \geq \beta^T(X_j^T)|v_i)$. Hence,

$$\begin{aligned}\Pr(\beta^S(X_i^S) \geq \beta^T(X_j^T)) &= \int_{\mathcal{V}} \Pr(\beta^S(X_i^S) \geq \beta^T(X_j^T)|v_i) h_{\mathcal{V}}(v_i) dv_i \\ &= \int_{\mathcal{V}} \Pr(\beta^S(X_i^T) \geq \beta^T(X_j^T)|v_i) h_{\mathcal{V}}(v_i) dv_i \\ &= \Pr(\beta^S(X_i^T) \geq \beta^T(X_j^T)).\end{aligned}$$

This establishes statement 1., as learning about both value components leads to the same numerator and denominator in Equation 18.

Next, consider statement 2. I show that when bidder $i$ faces a $X_j^S$-type opponent, for all $x_j \in (0,1)$ we have $H^S(x_j|\beta^S, \beta^S, X_j^S) < H^T(x_j|\beta^S, \beta^S, X_j^S)$. As both bidders follow the same bidding function $\beta^S$, the event of a win of bidder $i$ translates into the event of having a higher signal than his opponent. I depict the cumulative distributions of the looser's signal as an integral over $s$ by exploiting conditional independence in Assumption CI.

The joint event of bidder $i$ winning when learning $X_i^S$ and bidder $j$ having a signal realization $X_j^S = x_j$ has density $\int_0^1 f^S(x_j|s)\left[1 - F^S(x_j|s)\right] h(s) ds$. If bidder $i$ instead learns about $X_i^T$, his signal does not depend on $S$. Then, the joint event of him winning and his opponent having a signal realization $X_j^S = x_j$ has density $\int_0^1 f^S(x_j|s)\left[1 - F^T(x_j)\right] h(s) ds = \left[1 - F^T(x_j)\right] \int_0^1 f^S(x_j|s) h(s) ds = \left[1 - F^T(x_j)\right] f^s(x_j)$. Due to Assumption A1, I drop the superscripts of the signal distributions in the following. For all $x_j \in (0,1)$, we have

$$\begin{aligned}H^S(x_j|\beta^S, \beta^S, X_j^S) &= \frac{1}{\Pr(X_i^S \geq X_j^S)} \int_0^{x_j} \int_0^1 f(\tilde{x}_j|s)(1 - F(\tilde{x}_j|s)) h(s) ds d\tilde{x}_j. \\ H^T(x_j|\beta^S, \beta^S, X_j^S) &= \frac{1}{\Pr(X_i^T \geq X_j^S)} \int_0^{x_j} \int_0^1 f(\tilde{x}_j|s)(1 - F(\tilde{x}_j)) h(s) ds d\tilde{x}_j.\end{aligned}$$

Note that by Corollary 1, $\Pr(X_i^S \geq X_j^S) = \Pr(X_i^T \geq X_j^S) = \frac{1}{2}$. Hence,

$$H^S(x_j|\beta^S, \beta^S, X_j^S) - H^T(x_j|\beta^S, \beta^S, X_j^S) = \tag{19}$$

$$= 2 \int_0^{x_j} \int_0^1 f(\tilde{x}_j|s)(F(\tilde{x}_j) - F(\tilde{x}_j|s)) h(s) ds d\tilde{x}_j \tag{20}$$

$$= 2 \left[ \int_0^{x_j} F(\tilde{x}_j) \int_0^1 f(\tilde{x}_j|s) h(s) ds d\tilde{x}_j - \int_0^{x_j} \int_0^1 f(\tilde{x}_j|s) F(\tilde{x}_j|s)) h(s) ds d\tilde{x}_j \right] \tag{21}$$

$$= 2 \left[ \int_0^{x_j} F(\tilde{x}_j) f(\tilde{x}_j) d\tilde{x}_j - \int_0^{x_j} \int_0^1 f(\tilde{x}_j|s) F(\tilde{x}_j|s)) h(s) ds d\tilde{x}_j \right] \tag{22}$$

$$= 2 \left( \frac{F(x_j)^2}{2} - \int_0^1 \int_0^{x_j} f(\tilde{x}_j|s) F(\tilde{x}_j|s) d\tilde{x}_j h(s) ds \right) \tag{23}$$

$$= \left( F(x_j)^2 - \int_0^1 F(\tilde{x}_j|s)^2 h(s) ds \right). \tag{24}$$

By definition, it holds that $F(x_j) = \int_0^1 F(x_j|s) h(s) ds$. This and the strict Cauchy-Bunyakovsky-

Schwartz inequality yield for all $x_j \in (0,1)$,

$$F(x_j)^2 = \left[\int_s F(x_j|s)h(s)ds\right]^2 < \underbrace{\int_s h(s)ds}_{=1} \int_s F(x_j|s)^2 h(s)ds.$$

For all $x_j \in (0,1)$, the last inequality is strict, as $F(x_j|s)$ is not constant in the variable $s$ due to the strong MLRP.[33] This establishes that Equation 24 is negative for all $x_j \in (0,1)$.

Finally, consider statement 3. If $\beta^S(1) \leq \beta^T(0)$, expected payment against a $X_j^T$-type is trivially zero in DS and in CE with $(X_i^S, \beta^S)$. If $\beta^S(1) \leq \beta^T(0)$, Lemma 1. establishes that the expected payment in DS and the candidate equilibrium is also the same when facing a $X_j^T$-type opponent, *conditional* on a win. As in both cases, the winning probability is also the same due to independence, this also holds for the *unconditional* expected payment:

$$EP(X_i^S, \beta^S | X_j^T, \beta^T) = EP(\underbrace{X_i^T, \beta^S}_{DS} | X_j^T, \beta^T).$$

Statement 2. establishes that when facing a $X_j^S$-type opponent, the expected payment distribution conditional on a win with DS is dominated by the payment distribution of the candidate equilibrium after $X_i^S$. As by assumption, bidding function $\beta^S$ is increasing, FOSD implies a higher expected payment in the candidate equilibrium. Finally, as winning probability overall is the same in DS and the candidate equilibrium, this implies that the *unconditional* expected payment in DS is also lower than in the candidate equilibrium. Hence, we have

$$EP(X_i^S, \beta^S | X_j^S, \beta^S) > EP(\underbrace{X_i^T, \beta^S}_{DS} | X_j^S, \beta^S).$$

Therefore, overall expected payment in Equation 7 is strictly less under DS than after learning $X_i^S$ in the candidate equilibrium with $(X_i^S, \beta^S)$. □

***Proof of Theorem 1.*** Corollary 1 establishes the same expected gain and total winning probability in DS and CE. Lemma 3 establishes a strictly lower payment under DS than in CE. This rules out any $\rho^* > 0$ in equilibrium, and establishes the unique information selection $\rho^* = 0$ if an equilibrium exists.

The next steps establish existence. With $\rho^* = 0$, bidders are in an IPV setup. For fixed $\rho^* = 0$, it is a well known result that bidding $\beta^T(x) = \mathbb{E}\left[V_i | X_i^T = x\right]$ is an equilibrium in weakly dominant strategies. Whichever profitable deviation exists without information choice, will also exist in this setup with endogenous information selection. Thus, after learning $X_i^T$ and expecting the opponent to learn about $T_j$, above bidding function is a weakly dominant strategy.

Therefore, the only deviation we need to consider for bidder $i$ is to deviate and learn about common component. After seeing $X_i^S = x$, bidder $i$ is still in an IPV setup. If his opponent also learns

[33] This is because the Cauchy-Bunyakovsky-Schwartz inequality $\left[\int_a^b c(s)d(s)ds\right]^2 \leq \int_a^b c(s)^2 ds \cdot \int_a^b d(s)^2 ds$ is strict unless $c(s) = \alpha \cdot d(s)$ for some constant $\alpha$ (see Hardy et al., 1934, Chapter VI). In above argument, $c(s) = \sqrt{h(s)}$, and $d(s) = \sqrt{h(s)}F(x|s)$. Note that $F(x|s)$ is not constant in $s$ due to the strong MLRP unless $x \in \{0,1\}$.

about his private component, bidder $i$ has a weakly dominant strategy to bid his posterior valuation $\mathbb{E}\left[V_i|X_i^S = x\right]$. By Observation 2, for all $x$, $\mathbb{E}\left[V_i|X_i^S = x\right] = \mathbb{E}\left[V_i|X_i^T = x\right] = \beta^T(x)$. Hence, after deviating to the common component, bidder $i$ has the same best response after each signal realization, for any signal source. As $X_i^S$ and $X_i^T$ are distributed with equal marginal distribution $F(x)$ and are both independent from $X_j^T$ (which the opponent always learns in a candiadte equilibrium with $\rho^* = 0$), the deviating to component $S$ is not strictly profitable as it induces the same expected utility as the caniddate equilibrium with $\rho^* = 0$ when bidding optimally.

□

***Proof of Proposition 2.*** Let $\rho^* = 1$ with bidding function $\beta^S$ be a candidate equilibrium (CE). Fix the sum $I = S + T_i$ of the two components for bidder $i$.

Consider first the expected payment. Note that the proof of Proposition 1 holds step by step when instead of fixing $v_i$, the variable $I$ is fixed. That is, for all $I$,

$$\Pr(X_i^T \geq X_j^S|I) = \Pr(X_i^S \geq X_j^S|I) = \frac{1}{2}.$$

Holding the sum of the two components fixed, the winning probability in CE or in DS is unchanged for the case of two bidders. Therefore, Lemma 3 holds. Expected payment is strictly lower in DS than in the candidate equilibrium. This is because the proof of Lemma 3 does only rely on the bidding function $\beta^S$ being strictly increasing, not on any specific functional form. Therefore, varying the utility function does not change the observation that expected payment is strictly less under DS than in CE.

Next, consider the expected gain from DS. Given the sum $I$, a feasible common component realization lies in the interval $s \in [\underline{s}(I), \overline{s}(I)] := [\max\{I-1, 0\}, \min\{I, 1\}]$. Denote by $h(s|I) := \frac{h(s)h(I-s)}{h_{\mathcal{I}}(I)}$ the density of the common component conditional on $I$, where the density of the sum of the two components $I$ is $h_{\mathcal{I}}(I) = \int_0^1 h(s)h(I-s)ds$.

The cumulative distribution function of the common component $S$, conditional on $I$ and on bidder $i$ winning in the CE is for $s \in [\overline{s}(I), \underline{s}(I)]$:

$$\begin{aligned}
J^S(s|I) &:= \Pr(S \leq s | X_i^S \geq X_j^S, I) \\
&= \frac{1}{\Pr(X_i^S \geq X_j^S|I)} \int_{\underline{s}(I)}^{s} h(\tilde{s}|I) \underbrace{\int_0^1 f(x|\tilde{s})F(x|\tilde{s})dx}_{=\frac{1}{2}} d\tilde{s} \\
&= 2\int_{\underline{s}(I)}^{s} h(\tilde{s}|I)\frac{1}{2}d\tilde{s}.
\end{aligned}$$

$J^S(s|I) = 0$ for all $s \leq \underline{s}(I)$, where there exists no $T_i$ large enough to sum up to $I$. Furthermore, $J^S(s|I) = 1$ for all $s \geq I$.

Similarly, let the following be the cumulative distribution function of the common component $S$, conditional on $I$ and on bidder $i$ winning when following DS.

$$
\begin{aligned}
J^T(s|I) &= \Pr(S \le s | X_i^T \ge X_j^S, I) \\
&= \frac{1}{\Pr(X_i^T \ge X_j^S | I)} \int_{\underline{s}(I)}^{s} h(\tilde{s}|I) \int_0^1 f(x|I-\tilde{s}) F(x|\tilde{s}) dx d\tilde{s} \\
&= 2 \int_{\underline{s}(I)}^{s} h(\tilde{s}|I) \underbrace{\int_0^1 f(x|I-\tilde{s}) F(x|\tilde{s}) dx}_{\triangle(s|I)} d\tilde{s}.
\end{aligned}
$$

As before, $J^T(s|I) = 0$ for all $s < \underline{s}(I)$ and $J^T(s|I) = 1$ for all $s \ge \overline{s}(I)$.

Next, I show that $J^S(s|I)$ is FOSD over $J^T(s|I)$. Take any $s \le \frac{I}{2}$. Note that the proof of Lemma 2 holds step-by-step, if conditioning on $I$ instead of $v_i$. By Lemma 2, $\triangle(s|I) \ge \frac{1}{2}$. Therefore, for all $s \le \frac{I}{2}$, we have $J^T(s|I) \ge J^S(s|I)$. Note that at $s = \overline{s}(I)$, $J^S(\overline{s}(I)|I) = J^T(\overline{s}(I)|I) = 1$, as no higher $S$ can feasibly occur if the sum of the two components is $I$. Assume by contradiction that there exists a $s' \in (\frac{I}{2}, \overline{s}(I))$ such that $J^T(s'|I) < J^S(s'|I)$. Then, again due to Lemma 2, $\triangle(s|I) < \frac{1}{2}$ for all $s \in (\frac{I}{2}, I]$. Therefore, if $J^T(s'|I) < J^S(s'|I)$, we must have $J^T(s''|I) < J^S(s''|I)$ for all $s'' > s'$. However, this contradicts $J^S(I|I) = J^T(I|I) = 1$. This establishes FOSD of $J^S$ over $J^T$: for all $s \in [\underline{s}(I), \overline{s}(I)]$, we have $J^T(s|I) \ge J^S(s|I)$.

Conditional on the sum of the two components being $I$ and bidder $i$ winning, his expected gain in the CE is:

$$
\int_{\underline{s}(I)}^{\overline{s}(I)} u(s, I-s) dJ^S(s|I).
$$

Note that with $V_i = S + T_i$, the above integral reduces to $u(s, I-s) = I$.

Conditional on the sum of the two components being $I$ and bidder $i$ winning, the expected gain from DS is:

$$
\int_{\underline{s}(I)}^{\overline{s}(I)} u(s, I-s) dJ^T(s|I).
$$

By assumption (property 3. of the generalized utility function), $u(s, I-s)$ is a non-increasing function. Thus, FOSD implies that $\int_0^1 u(s, I-s) j^S(s|I) ds \le \int_0^1 u(s, I-s) j^T(s|I) ds$. This establishes the result: DS leads to a weakly higher expected gain conditional on a win, the same probability of a win, and a strictly lower payment.

□

***Proof of Proposition 3.*** The cumulative distribution of the highest signal among $N-1$ bidders who all learn about the common component $S = s$ is

$$
G(y) := \Pr(Y_i^S \le y) = \int_0^1 F(y|s)^{N-1} h(s) ds.
$$

For $v_i = 0$, we have $s = 0$ and $t_i = 0$, bidder $i$'s signal follows density $f(x|0)$ for any information selection. The probability of winning for bidder $i$ is $\int_0^1 f(x_i|0)F(x|0)^{N-1}dx_i = \frac{1}{N}$. For $v_i = 2$ (i.e., $s = 1$ and $t_i = 1$), winning probability of bidder $i$ with any signal is $\int_0^1 f(x|1)F(x|1)^{N-1}dx = \frac{1}{N}$. This is because in those two extreme examples, the signal is equally distributed in signals $X_i^T$ and $X_i^S$.

Next, consider a total value for bidder $i$ at $v_i = v_i \in (0, 2)$. Define the feasible set of the common component by $\mathcal{S}(v_i)$, and let $\hat{s}(v_i)$ be the common component disecting this interval, as defined in the proof of Proposition 1. Similarly, let $h(s|v_i) := \frac{h(s)h(v_i-s)}{h_{\mathcal{V}}(v_i)}$. As before, we have$\int_{\mathcal{S}(v_i)} \frac{h(s)h(v_i-s)}{h_{\mathcal{V}}(v_i)}ds = 1$ and $h(s|v_i) = h(v_i - s|v_i)$.

First, consider the probability of bidder $i$ having the highest signal realization, if bidder $i$ observes the outcome of the experiment $X_i^S$ about the common component.

$$\begin{aligned}
\Pr(X_i^S \geq Y_i^S|v_i) &= \int_{\mathcal{S}(v_i)} \int_0^1 f(x|s)F(x|s)^{N-1}h(s|v_i)dxds \\
&= \int_{\mathcal{S}(v_i)} \underbrace{\left[\frac{1}{n}F(x|s)^N\right]_0^1}_{=1/N} h(s|v_i)ds \\
&= \frac{1}{N}\int_{\mathcal{S}(v_i)} h(s|v_i)ds = \frac{1}{N}.
\end{aligned}$$

Learning about the common component as all the other bidders yields a probability of $\frac{1}{N}$ of having the highest signal realization, for every realization of $v_i$ in this symmetric setup.

Next, consider the probability of bidder $i$ having the highest signal realization, if he learns $X_i^T$ and is the only bidder learning about his private component.

$$\Pr(X_i^T \geq Y_i^S|v_i) = \int_{\mathcal{S}(v_i)} \int_0^1 f(x|v_i - s)F(x|s)^{N-1}h(s|v_i)dxds. \tag{25}$$

I use the following abbreviation for clarity of presentation: $\lambda(s, x|v_i) := h(s|v_i)F(x|s)^{N-2}$. Then, the probability of having the highest signal with $X_i^T$ can be expressed as

$$\begin{aligned}
\Pr(X_i^T \geq Y_i^S|v_i) &= \int_{\mathcal{S}(v_i)} \int_0^1 f(x|v_i - s)F(x|s)\lambda(s, x|v_i)dxds \\
&= \int_{\mathcal{S}(v_i)} \int_0^1 \frac{N-1}{N} f(x|v_i - s)F(x|s)\lambda(s, x|v_i)dxds \\
&\quad + \int_{\mathcal{S}(v_i)} \int_0^1 \frac{1}{N} f(x|v_i - s)F(x|s)\lambda(s, x|v_i)dxds.
\end{aligned}$$

Integrating the inner integral of the second summand by parts yields

$$
\begin{aligned}
&\int_{\mathcal{S}(v_i)}\int_0^1 \frac{1}{N} f(x|v_i-s)F(x|s)\lambda(s,x|v_i)dxds \\
=&\int_{\mathcal{S}(v_i)}\frac{1}{N}\int_0^1 f(x|v_i-s)F(x|s)^{N-1}dxh(s|v_i)ds \\
=&\int_{\mathcal{S}(v_i)}\frac{1}{N}\left(\underbrace{\left[F(x|v_i-s)F(x|s)^{N-1}\right]_0^1}_{=1.}-\int_0^1 (N-1)f(x|s)F(x|s)^{N-2}F(x|v_i-s)dx\right)h(s|v_i)ds \\
=&\frac{1}{N}\underbrace{\int_{\mathcal{S}(v_i)}h(s|v_i)ds}_{=1.}-\int_{\mathcal{S}(v_i)}\int_0^1\frac{N-1}{N}f(x|s)F(x|s)^{N-2}F(x|v_i-s)h(s|v_i)dxds \\
=&\frac{1}{N}-\int_{\mathcal{S}(v_i)}\int_0^1\frac{N-1}{N}f(x|s)F(x|v_i-s)\lambda(s,x|v_i)dxds.
\end{aligned}
$$

Plugging this back into equation 26 gives the following expression:

$$
\Pr(X_i^T \geq Y_i^S|v_i) = \tag{26}
$$

$$
\frac{1}{N}+\int_{\mathcal{S}(v_i)}\int_0^1\frac{N-1}{N}\left[f(x|v_i-s)F(x|s)-f(x|s)F(x|v_i-s)\right]\lambda(s,x|v_i)dxds. \tag{27}
$$

I show that the second summand in equation 27 is non-negative. For clarity of presentation, define $\mu(s,x|v_i) \coloneqq f(x|v_i-s)F(x|s)-f(x|s)F(x|v_i-s)$. Plugging in this notation and changing the order of integration in equation 27 yields

$$
\int_{\mathcal{S}(v_i)}\int_0^1\frac{N-1}{N}\left[f(x|v_i-s)F(x|s)-f(x|s)F(x|v_i-s)\right]\lambda(s,x|v_i)dxds \tag{28}
$$

$$
=\int_0^1\int_{\mathcal{S}(v_i)}\frac{N-1}{N}\mu(s,x|v_i)\lambda(s,x|v_i)dsdx \tag{29}
$$

$$
=\frac{N-1}{N}\int_0^1\left[\int_{\max\{v_i-1,0\}}^{\hat{s}(v_i)}\mu(s,x|v_i)\lambda(s,x|v_i)ds+\int_{\hat{s}(v_i)}^{\min\{v_i,1\}}\mu(s,x|v_i)\lambda(s,x|v_i)ds\right]dx. \tag{30}
$$

Note that the second summand can be rewritten as

$$
\begin{aligned}
\int_{\hat{s}(v_i)}^{\min\{v_i,1\}}\mu(s,x|v_i)\lambda(s,x|v_i)ds &= \int_{\max\{v_i-1,0\}}^{\hat{s}(v_i)}\mu(v_i-s,x|v_i)\lambda(v_i-s,x|v_i)ds \\
&= -\int_{\max\{v_i-1,0\}}^{\hat{s}(v_i)}\mu(s,x|v_i)\lambda(v_i-s,x|v_i),
\end{aligned}
$$

where the first step was by changing the label of the integration variable, and the second line followed from $\mu(s,x|v_i)=-\mu(v_i-s,x|v_i)$. Plugging this back into equation 30 yields:

$$\frac{N-1}{N}\int_0^1\int_{\max\{v_i-1,0\}}^{\hat{s}(v_i)}\mu(s,x|v_i)\left[\lambda(s,x|v_i)-\lambda(v_i-s,x|v_i)\right]dsdx.$$

Consider the expression in the square brackets in the inner integral first,

$$\lambda(s,x|v_i)-\lambda(v_i-s,x|v_i)=h(s|v_i)\left(F(x|s)^{N-2}-F(x|v_i-s)^{N-2}\right).$$

For $N=2$, the expression above is zero, as the term in the brackets is zero for any $s,x$ or $v_i$, which establishes the theorem for two bidders: winning probability in equation 27 is $\frac{1}{2}$.

For $N>2$, the strong MLRP and thus, FOSD[34] imply: for all $a<b$ and for all $x\in(0,1)$, we have $F(x|a)>F(x|b)$. As the integral is below $\hat{s}(v_i)$, we have $s<v_i-t$. Therefore, for $x\in(0,1)$:

$$\lambda(s,x|v_i)-\lambda(v_i-s,x|v_i)>0.$$

Furthermore, note that $\mu(s,x|v_i)\geq 0$ is a reverse hazard rate condition $f(x|v_i-s)F(x|s)-f(x|s)F(x|v_i-s)\geq 0$. A well-known implication of the MLRP is that for all $a<b$, we have reverse hazard rate dominance

$$\frac{f(x|a)}{F(x|a)}\leq\frac{f(x|b)}{F(x|b)}.$$

Due to $s<v_i-s$, it immediately follows that $\mu(s,x|v_i)\geq 0$ in the entire domain of integration. This establishes the non-negativity in the second summand of equation 27. Thus, for $N>2$ and $x\in(0,1)$ we have $\Pr(X_i^T\geq Y_i^S|v_i)>\frac{1}{N}$.

□

***Proof of Lemma 4.*** As bidders follow the same bidding function $\beta_f^T$ in the candidate equilibrium and in $\overline{DS}^f$, after any information choice a bidder wins if and only if he has the highest signal realization.

In the candidate equilibrium, there are four possibilities for bidder $i$:

1. $S=T_i=T_j$ with probability $\epsilon^2$ (denote the observed signals $X_i^{T=S}$ and $X_j^{T=S}$),
2. $T_i\neq S\neq T_i$ with probability $(1-\epsilon)^2$ (denote the observed signals $X_i^{T\neq S}$ and $X_j^{T\neq S}$),[35]
3. $T_i=S\neq T_j$ with probability $\epsilon(1-\epsilon)$,
4. $T_i\neq S=T_j$ with probability $\epsilon(1-\epsilon)$.

Consider the winning probability of bidder $i$ conditional on $v_i$ in each of those four possibilities. In possibility 1., if $V_i=v_i$, this implies that $S=T_i=v_i/2$.

$$\Pr(X_i^{T=S}\geq X_j^{T=S}|v_i)=\int_0^1 f(x_i|S=v_i/2)F(x_i|S=v_i/2)dx_i=\frac{1}{2}.$$

[34]For implications of the MLRP, like FOSD and reverse hazard rate dominance, see Milgrom and Weber (1982).

[35]Note that the probability of both components $S$ and $T_i$ being drawn independently but having the same realization has zero probability as the distribution of each component has no mass points.

In possibility 4., the winning probability is:

$$\Pr(X_i^{T\neq S} \geq X_j^{T=S}|v_i) = \int_0^1 \int_0^1 f(x_i|v_i - s)F(x_i|s)dx_i \frac{h(s)h(v_i - s)}{h_{\mathcal{V}}(v_i)}ds = \frac{1}{2}.$$

The last equality follows from the proof of Proposition 1 (it is the same equation as Equation 25) for the case of two bidders.

Furthermore, note that winning probabilities conditional on a win in possibility 2. and 3. are the same, as the following shows:

$$\begin{aligned}
\Pr(X_i^{T=S} \geq X_j^{T\neq S}|v_i) &= \int_0^1 \int_0^1 f(x_i|s)F(x_i)dx_i \frac{h(s)h(v_i - s)}{h_{\mathcal{V}}(v_i)}ds \\
\Pr(X_i^{T\neq S} \geq X_j^{T\neq S}|v_i) &= \int_0^1 \int_0^1 f(x_i|t)F(x_i)dx_i \frac{h(t)h(v_i - t)}{h_{\mathcal{V}}(v_i)}dt.
\end{aligned}$$

Therefore, in the candidate equilibrium, total winning probability conditional on $v_i$ is:

$$\begin{aligned}
&\underbrace{\left(\epsilon^2 + \epsilon(1-\epsilon)\right)}_{\text{1. and 4.}} \frac{1}{2} + \underbrace{\left((1-\epsilon)^2 + \epsilon(1-\epsilon)\right)}_{\text{2. and 3.}} \Pr(X_i^{T=S} \geq X_j^{T\neq S}|v_i) \\
&\qquad = \frac{\epsilon}{2} + (1-\epsilon)\Pr(X_i^{T=S} \geq X_j^{T\neq S}|v_i).
\end{aligned}$$

If the bidder deviates to $\overline{DS}^f$ instead, he always observes a signal $X_i^S$ based on the realization of $S$. For his opponent, there are two possibilities: either his opponent' private component is $T_j \neq S$ with probability $(1-\epsilon)$, or it is $T_j = S$ with probability $\epsilon$. Winning probabilities in both cases conditional on $v_i$ are:

$$\begin{aligned}
Pr(X_i^S \geq X_j^{T=S}|v_i) &= \int_0^1 f(x_i|S = v_i/2)F(x_i|S = v_i/2)dx_i = \frac{1}{2}. \\
\Pr(X_i^S \geq X_j^{T\neq S}|v_i) &= \int_0^1 \int_0^1 f(x_i|s)F(x_i)dx_i \frac{h(s)h(v_i - s)}{h_{\mathcal{V}}(v_i)}ds = \Pr(X_i^{T=S} \geq X_j^{T\neq S}|v_i).
\end{aligned}$$

Therefore, total winning probability of bidder $i$ conditional on $\overline{DS}^f$ is also

$$\frac{\epsilon}{2} + (1-\epsilon)\Pr(X_i^{T=S} \geq X_j^{T\neq S}|v_i).$$

This establishes that winning probability is equal at every $v_i$ in the candidate equilibrium and $\overline{DS}^f$. Total expected gain is:

$$\int_{\mathcal{V}} v_i h_{\mathcal{V}} \Pr(\text{i wins}|v_i)dv_i$$

Therefore, overall expected gain in the candidate equilibrium and in the $\overline{DS}^f$ is the same. □

***Proof of Lemma 5.*** Consider the distribution of signals of bidder $i$ conditional on winning in the candidate equilibrium, i.e. the distribution of the first order statistic. For the same four possibilities as in Lemma 4, we have the following distributions:

1. $M(x_i|T_i = S, T_j = S) := \Pr(X_i \le x_i | X_i^T \ge X_j^T, T_i = S = T_j) = H^S(x_i|\beta_f^S, \beta_f^S, X_j^S)$,
2. $M(x_i|T_i \ne S, T_j \ne S) := \Pr(X_i \le x_i | X_i^T \ge X_j^T, T_i \ne S \ne T_j) = H^T(x_i|\beta_f^S, \beta_f^S, X_j^T)$,
3. $M(x_i|T_i = S, T_j \ne S) := \Pr(X_i \le x_i | X_i^T \ge X_j^T, T_i = S \ne T_j) = H^S(x_i|\beta_f^S, \beta_f^S, X_j^T)$,
4. $M(x_i|T_i \ne S, T_j = S) := \Pr(X_i \le x_i | X_i^T \ge X_j^T, T_i \ne S = T_j) = H^T(x_i|\beta_f^S, \beta_f^S, X_j^S)$.

The last inequalities followed by definition of $H^K$ as defined in the main part of section 6.1. Due to independence between the signals of the bidders, and the same marginal distribution of both signals of $i$, we have

$$H^T(x_i|\beta_f^S, \beta_f^S, X_j^T) = H^S(x_i|\beta_f^S, \beta_f^S, X_j^T).$$

Furthermore, as established in the main text of section 6.1 in Inequality 14, increasing correlation decreases the first order statistic. That is, for all $x_i \in (0,1)$, we have

$$H^T(x_i|\beta_f^S, \beta_f^S, X_j^S) < H^S(x_i|\beta_f^S, \beta_f^S, X_j^S).$$

Hence, the overall distribution of the first oder statistic in the candidate equilibrium is

$$\begin{aligned} M^C(x_i) = & \underbrace{\epsilon^2}_{1.} H^S(x_i|\beta_f^S, \beta_f^S, X_j^S) + \underbrace{\epsilon(1-\epsilon)}_{4.} H^T(x_i|\beta_f^S, \beta_f^S, X_j^S) \\ & + \underbrace{\big((1-\epsilon)^2 + \epsilon(1-\epsilon)\big)}_{2.\text{ and } 3.} H^T(x_i|\beta_f^S, \beta_f^S, X_j^T). \end{aligned}$$

If bidder $i$ instead plays $\overline{DS}^f$, he observes always a signal about $X_i^S$. With probability $\epsilon$, his distribution in case of a win is $H^S(x_i|\beta_f^S, \beta_f^S, X_j^S)$ (if his opponent's private and common component are the same), and with probability $(1-\epsilon)$, his distribution in case of a win is $H^S(x_i|\beta_f^S, \beta_f^S, X_j^T)$. Thus, his overall distribution of his signal conditional on winning is

$$M^{DS}(x_i) = \epsilon H^S(x_i|\beta_f^S, \beta_f^S, X_j^S) + (1-\epsilon) H^S(x_i|\beta_f^S, \beta_f^S, X_j^T).$$

In Inequality 14 I establish that for all $x_i \in (0,1)$, we have $H^T(x_i|\beta_f^S, \beta_f^S, X_j^S) < H^S(x_i|\beta_f^S, \beta_f^S, X_j^S)$. This in turn implies that for all $x_i \in (0,1)$, the distribution of winning bids under the candidate equilibrium is FOSD over $\overline{DS}^f$.

$$M^{DS}(x_i) \ge M^C(x_i).$$

Therefore, expected payment is strictly higher under the candidate equilibrium than in $\overline{DS}^f$. □

***Proof of Proposition 4.*** The proof follows by combining the following two results as described in the main text. Lemma 4 shows that winning probability and expected gain from the deviation strategy $\overline{DS}^f$ is in the deviation strategy and in the candidate equilibrium. Lemma 5 establishes that $\overline{DS}^f$ leads to a strictly lower expected payment. Hence, DS is a strictly profitable deviation. □

***Proof of Proposition 5.*** The proof is by contradiction. I show that expected payoff from $DS^a$ is higher than in a CE with $\rho^* = 1$. Assume that in the candidate equilibrium, $\rho^* = 1$ and bidders follow a strictly increasing pure bidding function $\beta_a^S(x)$. Denote by $Y_i$ the highest signal realization of all bidders but bidder $i$.

The expected payment of bidder $i$ in the CE is:

$$\int_0^1 \beta_a^S(x_i) f^S(x_i) dx_i.$$

The expected payment of bidder $i$ from the $DS^a$ is:

$$\int_0^1 \beta_a^S(x_i) f^T(x_i) dx_i.$$

Due to symmetry, for all $x \in [0,1]$, we have $f^T(x_i) = f^S(x_i)$ according to Observation 1. Hence, the expected payment in the candidate equilibrium is the same as in $DS^a$.

Next, consider the expected gain from participating in the auction. Fix a value $v_i$ for bidder $i$, as in the preceding sections. In CE and in $DS^a$, bidder $i$ wins if he has the highest signal realization, as in both, bidders follow the same bidding function $\beta_a^S(\cdot)$. Formally, the winning probability of bidder $i$ for a fixed value $v_i$ under both regimes is:

$$\begin{aligned} \text{CE:} \qquad & \Pr(\beta_a^S(X_i^S) \geq \beta_a^S(Y_i^S)|v_i) = \underbrace{\Pr(X_i^S \geq Y_i^S|v_i)}_{\star^A}, \\ DS^a: \qquad & \Pr(\beta_a^S(X_i^T) \geq \beta_a^S(Y_i^S)|v_i) = \underbrace{\Pr(X_i^T \geq Y_i^S|v_i)}_{\star^{AA}}. \end{aligned}$$

For $N > 2$, by Proposition 3, the probability of a win is strictly higher with $DS^a$ than with CE for all $v_i \in (0,1)$. Hence, $\star^{AA} > \star^A$ for all $v_i \in (0,2)$. Winning probability in $DS^a$ is strictly higher than in CE for almost all $v_i$, for the same expected payment. $DS^a$ is a strictly profitable deviation.

□

***Proof of Lemma 6.*** Fix $\rho^* = 0$. This is the standard symmetric IPV setting, with the bidding function in the main text being a best response if all other bidders follow it. That is, given fixed $\rho^* = 0$, this bidding function consitutes a best response for both bidders.

The only part of the proposition left to be shown, is that no bidder has a profitable deviation that involves a different information choice variable $\rho_i$. Consider bidder $i$ deviating to $\rho_i \neq 0$. Note that the expected utility is a linear combination of the payoff after observing $X_i^S$ with probability $\rho_i$, and $X_i^T$ with probability $(1-\rho_i)$. Therefore, it suffices to consider the case $\rho_i = 1$ and showing that it does not lead to a strictly higher payoff than $\rho_i = 0$.

Due to the Independence Assumption IN, neither $X_i^T$ nor $X_i^S$ contain information about the opponent's signal due to Assumption IN. Furthermore, the value of the object conditional on a win does not depend on bidder $i$'s information choice due to Observation 2 and the irrelevance of the opponent's information. Therefore, as the choice of $\rho_i$ impacts neither the joint distribution, nor expected valuation conditional on a win, each bidder is indifferent between each $\rho_i \in [0,1]$ and plays the same best response after any signal realization (no matter its source). Therefore, the classic equilibrium of the all-pay auction is also an equilibrium of this game that involves information selection. □

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