Preliminary version!

Individual Probabilities of Unemployment under Automation*

Hideki Nakamura, Osaka City University
Masakatsu Nakamura, Rissho University
Joseph Zeira, Hebrew University of Jerusalem

Abstract: This study considers mismatch unemployment under automation. We elucidate the individual probabilities of unemployment (IPU) of workers who have their own unsuited job categories. Given labor demand of an economy, regardless of a decrease in the availability of job categories caused by automation, the mean of IPU which is equal to the macroeconomic unemployment rate remains unchanged. However, the variance of IPU does increase, that is, more heterogeneity of workers can reveal. Examining the dynamics of IPU, job mismatch can be severe with the possibility of unemployment persistence.

Keywords: Mismatch unemployment; Automation; Individual mismatch rates; Individual probabilities of unemployment (IPU); Unemployment persistence.

JEL Classification: E23, J64, O33.

*Corresponding author: Hideki Nakamura, Faculty of Economics, Osaka City University 3-3-138 Sugimoto, Sumiyoshi, Osaka 558-8585, Japan, Phone and fax:+81-6-6605-2271, E-mail: hnakamur@econ.osaka-cu.ac.jp

1. Introduction

The motivation of this study is based on the labor movement caused by automation. Routine occupations disappear because of automation. It is now possible to have automation in the services industries, because robots with artificial intelligence (AI) can replace manual labor requiring flexible interaction. The Nomura Research Institute and Oxford University reported that in 10 to 20 years, robots could handle 235 of the 601 types of jobs studied. Consequently, it causes controversy, as there is uncertainty whether machines will take human jobs.

This study explores mismatch unemployment under automation. We consider individual mismatch rates of workers who have their own unsuited job categories. Examining a random matching between jobs and workers as the benchmark, we elucidate their individual probabilities of unemployment (IPU). Given labor demand of an economy, regardless of a decrease in the availability of job categories, the mean of IPU which is equal to the macroeconomic unemployment rate remains unchanged. However, the variance of IPU does increase, that is, more heterogeneity of workers can reveal. Examining the dynamics of IPU, job mismatch can be severe with the possibility of unemployment persistence.

Our model has two distinguished features. One is the heterogeneity of workers that is represented by their own suitable and unsuitable job categories. Individuals would differently have their own suitable and unsuitable job categories because of their innate difference and different types of education, regardless of their abilities.[1] The second feature of our model is the discrimination between the individual effect caused by their heterogeneity and the macroeconomic effect caused by labor demand of an economy. The IPU depend on the individual mismatch rates and labor demand which is exogenously given. The individual mismatch rates represent the heterogeneity of individuals who differently have their suitable and unsuitable jobs. We represent labor demand by the number of interviews or intern. Thus, we can distinguish between the individual factor and the macroeconomic factor. Given the two features, we decompose the macroeconomic unemployment rate into the IPU that depend on the degree of automation and the number of interviews:

$$\frac{U}{L} = \sum_x \frac{U(x;\gamma,v)}{L},$$

where U is the number of unemployed workers, L is the total number of workers, and thus, U/L represents the macroeconomic unemployment rate. x represents the difference in unsuitable jobs among workers. γ represents the degree of automation, and v is the number of interviews per a worker.

This study belongs to three lines of research. The first line is the effect of automation. Many studies focus on the effects on wages, labor income share, job polarization, and economic growth (see, Zeira, 1998; Acemoglu, 2010; Autor and Dorn,

[1] Individuals would have their own suitable and unsuitable job categories according to their own characters and education of vocational schools and majors at universities. In Section 4, we extend our model by considering workers with different abilities.

2013; Jaimovich and Siu, 2014; Alensia, Battisti, and Zeira, 2016). Automation would differently affect employment of workers because of their different job opportunities. Thus, we consider the IPU of workers, which lie behind the macroeconomic unemployment rate. We elucidate how heterogeneity among workers appears under automation.[2]

The second line is the empirical studies about mismatch unemployment and unemployment persistence (see Arulampalam, Booth, and Taylar, 2000; Mukoyama and Sahin, 2009; and Sahin, Song, Topa, and Violante, 2014). Examining the dynamics of IPU, we show that workers who currently have a large IPU may have a high possibility of unemployment in the next period, which implies the possibility of unemployment persistence. Furthermore, the job mismatch will be severe because the progress of automation increases the variance of IPU.[3]

The third line this study touches is the analysis of macroeconomic variables with heterogeneous individuals. Yoshikawa (2015) criticize micro-founded models with representative agents because those models may obscure the accurate dynamics of aggregate variables. This study sheds light on the heterogeneity of workers which is represented by their suitable and unsuitable jobs and examines the macroeconomic unemployment rate.[4]

The remainder of the paper is organized as follows. In Section 2, we explain an example and then a general model. We detail individual mismatch rates and IPU. In Section 3, we examine the dynamics of IPU distribution. We conclude our paper with a brief summary in Section 4.

2. Individual mismatch rates and IPU

2.1 Examples

In this subsection, we present intuitive examples. We first consider two job categories 1 and 2 with the number of workers, represented as L ($L \geq 2$). Those workers randomly have their own suited and unsuited job categories.[5] The number of unsuited job categories is assumed to be unity for any worker. See Figure 1. Thus, half of the workers are suited for job 1, but unsuited for job 2, while the remaining workers are suited for job 2, but unsuited for job category 1. That is, we have two types of workers with respect to suited and unsuited jobs, (G, B) and (B, G), where

[2] The technical change represented by automation may create some new jobs. However, we could experience a decrease in the number of human jobs by AI. See Hrdy (2017). Furthermore, even with an increase in the number of jobs, those would include unsuited jobs for some workers.

[3] Nakamura and Zeira (2017) endogenously considered the macroeconomic unemployment and examined mismatch unemployment under automation. Automation can reduce the macroeconomic unemployment rate when capital inputs increase sufficiently. However, the variance of IPU can increase, even with the decrease in the macroeconomic unemployment rate.

[4] In Appendix C, we examine an augmented model with two types of job categories and two types of individuals.

[5] In our model, the perception of workers about their suited and unsuited jobs has no effect on the result. Even when workers know their suited and unsuited jobs, we consider a random job matching.

G and B imply being good and bad at the i-th job ($i = 1, 2$). The population ratios of the two groups are $\frac{1}{2}$.

To start, we consider the case of no mechanized jobs, that is, both jobs are available, to which workers are assigned randomly. We define the individual rate of mismatch as $phi(x)$ in which x represents the number of unsuited jobs. We can represent the individual mismatch rates of the two groups, (G, B) and (B, G) as:

$$\phi(1) = \frac{1}{2}.$$

The density function of x is represented as $f(x)$. We have $f(1) = 1$.

We define the mean and variance of the individual mismatch rates as follows:

$$\mu_\phi \equiv \sum_x \phi(x)f(x) \;\; and \;\; \sigma_\phi^2 \equiv \sum_x [\phi(x) - \mu_\phi]^2 f(x). \tag{1}$$

The mean of individual mismatch rates for the groups, (G, B) and (B, G) is represented as: $\mu_\phi = \phi(1)f(1) = \frac{1}{2}$. There is no variance for the individual mismatch rates: $\sigma_\phi^2 = 0$. Thus, given no automation, the heterogeneity among workers is concealed.

Each type of workers which represented as x can have the number of interviewees, V. We obtain the expected value of employees: $E(H) = (1 - \phi(1))V$ in which H is the number of employees of each type. Thus, the expected employment rate can be represented as $\frac{E(H)}{L}$.

We now define the IPU:

$$\rho(1; v) \equiv 1 - \frac{E(H)}{L} = 1 - (1 - \phi(1))v, \qquad (2$$

where we define: $v \equiv \frac{V}{L}$. Thus, v is the number of interviews per a worker. Hereafter, we represent $\rho(x; v$ as $\rho(x)$.

An increase in the individual mismatch rates increases the IPU. However, an increase in the number of interviews decreases the IPU. When $v = 1$, the IPU of (G, B) and (B, G) is equal to the individual mismatch rate: $\rho(1) = \phi(1)$. When $v = 2$, we have: $\rho(1) = 0$. That is, there is no unemployment when workers can recieve interviews not of job category 1 but also 2.

The mean and variance of IPU are defined as follows:

$$\mu_\rho \equiv \sum_x \rho(x)f(x) \;\; and \;\; \sigma_\rho^2 \equiv \sum_x [\phi(x) - \mu_\rho]^2 f(x). \tag{3}$$

We have: $\mu_\rho = \rho(1)f(1) = 1 - \frac{1}{2}v$ and $\sigma_\rho^2 = 0$.

We now assume that job 1 is mechanized, that is, only job 2 is available to workers (see Figure 1). The workers of type (G, B) are unemployed because their suited job 1 is mechanized. The workers of type (B, G) are employed because their unsuited job 1 is mechanized. Thus, the IPU for the two groups, $x = 1$ and $x = 0$ are, respectively represented as follows:

$$\phi(1) = 1 \;\; and \;\; \phi(0) = 0.$$

We have the following densities:

$$f(1) = \frac{1}{2} \;\; and \;\; f(0) = \frac{1}{2}.$$

The mean of IPU remains unchanged under automation:

$$\mu_\phi = \sum_{x=0}^{1} \phi(x) f(x) = \frac{1}{2}.$$

The variance of IPU increases because of automation:

$$\sigma_\phi^2 = \sum_{x=0}^{1} [\phi(x) - \mu_\phi]^2 f(x) = \frac{1}{4}.$$

Automation reveals the heterogeneity among workers.

Because there exists only job 1, we assume that $v \leq 1$. We have the IPU as follows:

$$\rho(1) = 1 \;\; and \;\; \rho(0) = 1 - v.$$

Thus, the mean and variance of IPU are represented as:

$$\mu_\rho = \sum_{x=0}^{1} \rho(x) \frac{1}{2} = 1 - \frac{1}{2} v.$$

$$\sigma_\rho^2 = \sum_{x=0}^{1} [\phi(x) - \mu_\rho]^2 \frac{1}{2} = \sigma_\phi^2 v^2 = \frac{1}{4} v^2.$$

Next, consider three different jobs, 1, 2, and 3. By assuming that the number of unsuited jobs equals unity, we examine the following three types: (G, G, B), (G, B, G), and (B, G, G). The individual mismatch rate is represented as:$\phi(1) = \frac{1}{3}$ and $f(1) = 1$. We have the following mean and variance of individual mismatch rates: $\mu_\phi = \frac{1}{3}$ and $\sigma_\phi^2 = 0$.

The IPU is represented as:

$$\rho(1) = 1 - (1 - \phi(1))v = 1 - \frac{2}{3} v.$$

When $v = 1$, the IPU is equal to the individual mismatch rate: $\rho(1) = \phi(1)$. When $v = \frac{3}{2}$, we have: $\rho(1) = 0$, that is, there is no unemployment. The mean and variance are, respectively represented as: $\mu_\rho = 1 - \frac{2}{3} v$ and $\sigma_\rho^2 = 0$.

We now assume that job 1 is mechanized, that is, jobs 2 and 3 are available for workers. We can represent the IPU of (G, G, B), (G, B, G), and (B, G, G) as follows:

$$\phi(1) = \frac{1}{2}, \;\; \phi(0) = 0,$$

with the densities: $f(1) = \frac{2}{3}$ and $f(0) = \frac{1}{3}$.

The mean of individual mismatch rates remains unchanged under automation while the variance increases:

$$\mu_\phi = \sum_{x=0}^{1} \phi(x) f(x) = \frac{1}{3},$$

$$\sigma_\phi^2 = \sum_{x=0}^{1} [\phi(x) - \mu_\phi]^2 f(x) = \frac{1}{18}.$$

The IPU are represented as:

$$\rho(1) = 1 - \frac{1}{2}v, \quad \rho(0) = 1 - v.$$

Thus, we have the following mean and variance of IPU:

$$\mu_\rho = \sum_{x=1}^{2} \rho(x) f(x) = 1 - \frac{2}{3}v.$$

$$\sigma_\rho^2 = \sum_{x=1}^{2} [\rho(x) - \mu_\rho]^2 f(x) = \frac{1}{18}v^2.$$

Given the number of interviews, the mean of IPU remains unchanged while the variance increases. That is, more heterogeneity of workers reveals under automation.

Finally, we assume that jobs 1 and 2 are mechanized, that is, the remaining job 3 is available for workers. We have the following individual mismatch rates:

$$\phi(1) = 1, \quad \phi(0) = 0,$$

with the densities, $f(1) = \frac{1}{3}$ and $f(0) = \frac{2}{3}$. Thus, the mean and variance of individual mismatch rates are represented as:

$$\mu_\phi = \sum_{x=0}^{1} \phi(x) f(x) = \frac{1}{3},$$

$$\sigma_\phi^2 = \sum_{x=0}^{1} [\phi(x) - \mu_\phi]^2 f(x) = \frac{2}{9}.$$

While the mean of individual mismatch rates remains unchanged, the variance increases.

The IPU are represented as:

$$\rho(1) = 1, \quad \rho(0) = 1 - v.$$

Given the number of interviews, the mean of IPU remains unchanged under automation:

$$\mu_\rho = \sum_{x=0}^{1} \rho(x) f(x) = 1 - \frac{2}{3}v.$$

However, the variance increases further, that is, additional heterogeneity among the three types of workers appears because of the progress in automation:

$$\sigma_\rho^2 = \sum_{x=0}^{1} [\rho(x) - \mu_\rho]^2 f(x) = \frac{2}{9} v^2.$$

2.2 General model

We consider a large number of heterogeneous jobs, represented as N with the number of workers, L ($L \geq N$), respectively (see Figure 2). Mechanized jobs are given exogenously. The number of job categories using machines is represented as Γ ($\Gamma < N$). Thus, $N - \Gamma$ represents the number of nonmechanized jobs that require workers. We make the following two important assumptions.

Workers can be categorized into groups by the number of jobs that workers are unsuited for and that machines cover, represented as x. In a group, workers have an equal number of unsuited jobs that machines cover, represented as x, which implies equal IPU. The IPU for a type x-th worker can be represented as follows:

$$\phi(x) \equiv \frac{b - x}{N - \Gamma}, \qquad (A1)$$

where $x = 0, 1, \cdots, b$.[6]

Thus, we can represent the mean of the individual mismatch rates as follows:

$$\mu_\phi \equiv \sum_{x=0}^{b} \rho(x) f(x), \qquad (4)$$

where $f(x)$ is the density function of x.

We now explore $f(x)$. The number of patterns of unsuited jobs, which is equal to the number of worker types, is represented as follows:

$$\begin{pmatrix} N \\ b \end{pmatrix}. \qquad (5)$$

We can represent the number of patterns where machines cover jobs that workers are unsuited for as follows:

$$\begin{pmatrix} \Gamma \\ x \end{pmatrix}, \qquad (6)$$

where $x = 0, 1, \cdots, b$.

Furthermore, the number of patterns where machines cannot cover jobs that workers are unsuited for is represented as:

$$\begin{pmatrix} N - \Gamma \\ b - x \end{pmatrix}, \qquad (7)$$

[6]When $\Gamma < m$, the maximum value of x is Γ.

where $x = 0, 1, \cdots, b$.

Thus, using (5)–(7), $f(x)$ follows the following hypergeometric distribution:

$$f(x) = \frac{\begin{pmatrix} \Gamma \\ x \end{pmatrix}\begin{pmatrix} N-\Gamma \\ b-x \end{pmatrix}}{\begin{pmatrix} N \\ b \end{pmatrix}}, \tag{8}$$

where $x = 0, 1, \cdots, b$.

The hypergeometric distribution implies the following mean, μ_x and variance, σ_x^2:

$$\mu_x \equiv \sum_{x=0}^{b} x f(x) = m\frac{\Gamma}{N} \quad and \quad \sigma_x^2 \equiv \sum_{x=0}^{b}(x-\mu_x)^2 f(x) = \frac{N-b}{N-1} b \frac{\Gamma}{N}(1-\frac{\Gamma}{N}). \tag{9}$$

We now examine the mean and variance of the IPU.

$$\mu_\phi \equiv \sum_{x=0}^{b} \phi(x) f(x) \quad and \quad \sigma_\phi^2 \equiv \sum_{x=0}^{b}[\phi(x)-\mu_\phi]^2 f(x). \tag{10}$$

Lemma 1: *(Distribution of IPU). Suppose Assumption (A1) holds. Using a hypergeometric distribution, the mean and variance of the individual mismatch rates are, respectively, represented as follows:*

$$\mu_\phi = \frac{b}{N} \quad and \quad \sigma_\phi^2 = \frac{\Gamma}{N-\Gamma}\frac{N-b}{N-1}\frac{b}{N^2}. \tag{11}$$

Proof: We obtain the mean of the individual mismatch rates:

$$\mu_\phi = \sum_{x=0}^{b}\phi(x) f(x) = \frac{b}{N-\Gamma}\sum_{x=0}^{b} f(x) - \frac{1}{N-\Gamma}\sum_{x=0}^{b} x f(x)$$

$$= \frac{b}{N-\Gamma} - \frac{\mu_x}{N-\Gamma} = \frac{b}{N}.$$

Furthermore, the variance of the individual mismatch rates can be represented as follows:

$$\sigma_\phi^2 = \sum_{x=0}^{b}[\phi(x)-\mu_\rho]^2 f(x) = \frac{1}{(N-\Gamma)^2}\sum_{x=0}^{b}[(b-x)-(N-\Gamma)\frac{N}{b}]^2 f(x)$$

$$= \frac{\sigma_x^2}{(N-\Gamma)^2} = \frac{\Gamma}{N-\Gamma}\frac{N-b}{N-1}\frac{b}{N^2}. \qquad \|$$

Regardless of automation, the mean of the individual mismatch rates remains to take $\frac{b}{N}$, which represents the ratio of the initial number of unsuited jobs to the total number of jobs (see Figure 3). That is, it is never affected by automation. While an increase in the number of mechanized jobs decreases the numbers of jobs that workers are both suited and unsuited for, these decreases cancel each other out.

However, an increase in the number of mechanized jobs increases the variance of the individual mismatch rates: $\frac{\partial \sigma_\phi^2}{\partial \Gamma} > 0$. There is a larger difference among workers in terms of the effect on their suited and unsuited jobs, because of a decrease in the total number of available jobs. That is, more heterogeneity among workers exists. As shown in Figure 3, $f(x)$ take high values with high individual mismatch rates.[7]

We examine the number of employees of each workers' type which is represented as H:

$$H(x) \sim B(\phi(x); V), \tag{12}$$

where the probability of the binomial distribution is represented as:

$$q(H(x); \phi(x), V) = \begin{pmatrix} V \\ H(x) \end{pmatrix} (1 - \phi(x))^{H(x)} \phi(x)^{V - H(x)}.$$

Note that V is the number of interviewees for x-th type. We obtain the expected value of $H(x)$: $E(H(x)) = (1 - \phi(x))V$.

We define the IPU. The IPU of the x-th type, $\rho(x)$ can be represented as follows:

$$\rho(x) \equiv 1 - \frac{E(H(x))}{L} = 1 - (1 - \phi(x))v. \tag{13}$$

We assume that $0 \leq \rho(x) \leq 1$.

An increase in the individual mismatch rate per an interview increases the IPU. Workers can obtain the opportunities of interviews more than one time.[8] Thus, the number of interviews in each of intermediate goods decreases the IPU. However, a decrease in the number of available job categories itself increase the IPU. Figure 3 illustrates the densities with the exes, $\frac{U(x)}{L} = 1 - \frac{H(x)}{L}$ and x. In the figure, given x, we have the binomial distribution with the mean $\rho(x)f(x)$. Under the law of large numbers, the binomial distribution represented on the axis, $\frac{U(x)}{L}$ spikes at its mean, $\rho(x)f(x)$. The variance in the IPU still exists because of the variance in x. The expected value of macroeconomic unemployment rates is equal to the sum of $\rho(x)f(x)$.

We now examine the mean and variance of the IPU.

$$\mu_\rho \equiv \sum_{x=0}^{b} \rho(x)f(x) \quad and \quad \sigma_\rho^2 \equiv \sum_{x=0}^{b} [\rho(x) - \mu_\phi]^2 f(x). \tag{14}$$

Lemma 2: *(Distribution of IPU). Suppose Assumptions (A1) and (A2) hold. Using a hypergeometric distribution, the mean and variance of the IPU are, respectively, represented as follows:*

$$\mu_\rho = 1 - (1 - \mu_\phi)v \quad and \quad \sigma_\rho^2 = \sigma_\phi^2 v^2. \tag{15}$$

[7] There can exist a difference between the mode and mean of $f(x)$.

[8] We assume that if they obtain the successful matching, they do not have interviews any more.

Proof: From (11), (13), and (14), we have: (15). ‖

Proposition 1: (IPU under automation) *Suppose Assumption (A1) holds. (a) Under automation, the mean of the IPU remains unchanged. However, the variance of the IPU does increase; that is, more heterogeneity among workers exists. (b) Given a large number of workers, the macroeconomic unemployment rate is equivalent with the mean of IPU.*

3. Dynamics of IPU

In this section, we consider the progress of automation from period 0 to period 1. We examine the dynamics of their IPU distribution between the two periods. We first consider two groups: one is the group of workers who cannot work in period 0 and the second group is workers who work in that period but lose their jobs because of automation.

We represent the number of mechanized jobs and total number of available jobs in period 0 as Γ_0 and N_0, respectively. We have the IPU:

$$\rho(x_0) = 1 - (1 - \phi(x_0))v,$$

where $\phi(x_0) = \frac{b - x_0}{N_0 - \Gamma_0}$. The mean and variance of IPU in period 0 are, respectively, represented as follows:

$$\mu_{\rho 0} = 1 - (1 - \mu_{\phi 0})v \quad \textit{and} \quad \sigma^2_{\rho 0} = \sigma^2_{\phi 0} v^2,$$

where $\mu_{\phi 0} = \frac{b}{N_0}$ and $\sigma^2_{\phi 0} = \frac{\Gamma_0}{N_0 - \Gamma_0} \frac{N_0 - b}{N_0 - 1} \frac{b}{N_0^2}$.

In the end of period 0, the workers are divided into the employed and unemployed workers. Furthermore, in the beginning of period 1, a part of workers who are employed in period 0 lose their jobs because of the progress in automation, represented as $\Delta\Gamma > 0$. Thus, the workers are divided into the following groups:

$$L = U_0 + E_0 = U_0 + M_0 + \bar{M}_0, \tag{16}$$

where $E_0 = M_0 + \bar{M}_0$. U_0 is the number of unemployed workers in period 0 and E_0 is the number of employed workers in period 0. M_0 is the number of workers in which their jobs are mechanized in period 1 and $\bar{M}_0$ is the number of workers in which their jobs are not mechanized in period 1.

Dividing (16) by L, we have:

$$1 = l(U_0) + l(E_0) = l(U_0) + l(M_0) + l(\bar{M}_0),$$

where $l(j)$ represents the ratio of j-th group to the total workers ($j = U_0$, E_0, M_0, and $\bar{M}_0$).

Given the law of large numbers, we obtain:

$$l(U_0) = \sum_{x_0=0}^{b} \rho(x_0) f(x_0) = 1 - (1 - \mu_{\phi 0})v = \mu_{\rho 0} \quad l(E_0) = 1 - \mu_{\rho 0}. \tag{17}$$

We also have:

$$l(M_0) = \frac{\Delta\Gamma}{N(\Gamma_0) - \Delta\Gamma} \sum_{x_0=0}^{b} (1 - \rho(x_0)) f(x_0) = \frac{\Delta\Gamma}{N(\Gamma_0) - \Delta\Gamma} (1 - \mu_{\rho 0}),$$

$$l(\bar{M}_0) = (1 - \frac{\Delta\Gamma}{N(\Gamma_0) - \Delta\Gamma})(1 - \mu_{\rho 0}).$$

When the mean of IPU is large, the ratio of unemployed workers is high because of their large IPU. However, the ratio of employed workers is low with the large mean of IPU. The ratio of workers who lose their jobs because of automation increases with the degree of automation.

We first see the IPU of the x_0-th group in that period. Any worker of the x_0-th group faces the following IPU in period 1:

$$\rho(\Delta x; x_0, \Gamma_0, \Delta\Gamma, v) = 1 - (1 - \phi(\Delta x; x_0, \Gamma_0, \Delta\Gamma))v,$$

where

$$\phi \Delta x; x_0, \Gamma_0, \Delta\Gamma) = \frac{b(x_0) - \Delta x}{N(\Gamma_0) - \Delta\Gamma},$$

$b(x_0) \equiv m - x_0$, $N(\Gamma_0) \equiv N_0 - \Gamma_0$, $\Delta\Gamma \geq 0$, and $\Delta x = 0, 1, \cdots, b(x_0)$. Hereafter, we denote $\rho(\Delta x; x_0)$.

The density function of Δx of the x_0-th group is represented as:

$$f(\Delta x | x_0, \Gamma_0, \Delta\Gamma) = \frac{\begin{pmatrix} \Delta\Gamma \\ \Delta x \end{pmatrix} \begin{pmatrix} N(\Gamma_0) - \Delta\Gamma \\ b(x_0) - \Delta x \end{pmatrix}}{\begin{pmatrix} N(\Gamma_0) \\ b(x_0) \end{pmatrix}}, \tag{18}$$

where $\Delta x = 0, 1, \cdots, b(x_0)$. Hereafter, we denote $f(\Delta x | x_0)$.

We now see the mean of IPU of two groups represented as U_0 and M_0 in period 1:

$$\mu_{\rho 1}(U_0) \equiv \frac{1}{l(U_0)} \sum_{x_0=0}^{b} \sum_{\Delta x=0}^{b(x_0)} \rho(\Delta x; x_0, U_0) f(\Delta x | x_0) \rho(x_0) f(x_0),$$

$$\mu_{\rho 1}(M_0) \equiv \frac{1}{l(M_0)} \sum_{x_0=0}^{b} \sum_{\Delta x=0}^{b(x_0)} \rho(\Delta x; x_0, M_0) f(\Delta x | x_0)(1 - \rho(x_0)) f(x_0) \frac{\Delta\Gamma}{N(\Gamma_0) - \Delta\Gamma}.$$

We have: $\rho(\Delta x; x_0) = \rho(\Delta x; x_0, j)$ $(j = U_0, M_0, \bar{M}_0)$.

The mean of IPU of group M_0 is equal to that of group E_0, that is, $\mu_{\rho 1}(M_0) = \mu_{\rho 1}(E_0)$:[9]

$$\mu_{\rho 1}(E_0) \equiv \frac{1}{l(E_0)} \sum_{x_0=0}^{b} \sum_{\Delta x=0}^{b(x_0)} \rho(\Delta x; x_0, E_0) f(\Delta x | x_0)(1 - \rho(x_0)) f(x_0).$$

[9]The mean of group E_0 is also equal to that of group $\bar{M}_0$. Furthermore, the variance of IPU of group E_0 is equal to those of groups M_0 and $\bar{M}_0$.

Note that $l(E_0)\frac{\Delta\Gamma}{N(\Gamma_0)-\Delta\Gamma} = l(M_0)$.

Lemma 2: *(Means of IPU for groups U_0 and M_0 in period 1). Suppose Assumption (A1) and holds. The expected values of IPU of groups U_0 and M_0 are respectively, represented as:*

$$\mu_{\rho 1}(U_0) = \mu_{\rho 0} + \frac{\sigma_{\rho 0}^2}{\mu_{\rho 0}}, \tag{19}$$

$$\mu_{\rho 1}(M_0) = \mu_{\rho 0} - \frac{\sigma_{\rho 0}^2}{1-\mu_{\rho 0}}. \tag{20}$$

Proof: We obtain:

$$\sum_{\Delta x=0}^{b(x_0)} \rho(\Delta x; x_0, U_0) f(\Delta x | x_0) = \rho(x_0).$$

This implies:

$$\mu_{\rho 1}(U_0) = \frac{1}{l(U_0)} \sum_{x_0=0}^{b} \rho(x_0)^2 f(x_0) = \mu_{\rho 0} + \frac{\sigma_{\rho 0}^2}{\mu_{\rho 0}}.$$

Furthermore, by use of $\mu_{\rho 1}(M_0) = \mu_{\rho 1}(E_0)$, we obtain:

$$\mu_{\rho 1}(M_0) = \frac{1}{l(E_0)} \sum_{x_0=0}^{b} \rho(x_0)(1-\rho(x_0)) f(x_0) = \frac{1}{l(E_0)}[\mu_{\rho 0} - (\sigma_{\rho 0}^2 + \mu_{\rho 0}^2)]$$

$$= \mu_{\rho 0} - \frac{\sigma_{\rho 0}^2}{1-\mu_{\rho 0}}. \qquad \|$$

We examine the IPU in period 1 of workers who belong to x_0-th group in period 0. Their IPU has a variance in period 1 because of the different effect of automation. Consequently, in period 1, the means of IPU in the two groups, U_0 and E_0 depend on the mean and variance in period 0. A large variance of IPU in period 0 increases the mean of IPU of group U_0 but decreases the mean of IPU of group E_0, that is, of group M_0. Furthermore, those effects are strengthened by the sizes of the groups. When the number of unemployed workers is small, that is, when the group size, $l(U_0)$ is small, it increases the mean of their IPU in period 1. A large group size, $l(E_0)$ implies a small mean of their IPU in period 1.

In Figure 4, by normalizing the group sizes, we illustrate the density functions of IPU in the two groups, U_0 and E_0.[10] The bold line represents the density function of the group U_0 which is given by $f(\Delta x|x_0, U_0)\frac{\rho(x_0)f(x_0;\Gamma_0)}{l(U_0)}$. The dotted line represents the density function of the group E_0 which is given by $f(\Delta x|x_0, U_0)\frac{(1-\rho(x_0))f(x_0;\Gamma_0)}{l(E_0)}$. A large variance of IPU in period 0 can cause the bipolarization in the two IPU

[10] As shown in the definition of $\mu_{\rho 1}(\cdot)$, these density functions are based on the three dimensional density function in which the horizontal axes are x and Δx and the vertical axis is $f(x)$. By summing x and Δx, we see the density functions in Figure 4.

distributions. Note that the variance of IPU in period 0 has the asymmetric effect on the two IPU distribution because of the different group sizes. Furthermore, the variances of IPU in the two distributions involve the effect of the progress in automation. The variances of the two groups with a rapid progress in automation.

Proposition 2: *(Dynamics of IPU for the two groups, U_0 and M_0). Suppose Assumption (A1) holds. (a) In period 1, workers who are unemployed in period 0, that is, workers in the group U_0 face a large mean of their IPU in period 1 because of the variance of IPU in period 0. It implies the possibility of unemployment persistence. (b) Even when workers can work in period 0, that is, even in the group M_0, some of them will have unemployment in period 1 as a result of the progress in automation. However, when the variance of IPU in period 0 is large, the mean of their IPU in period 1 is low.*

We next examine the IPU of an economy in period 1 via those of groups, U_0 and M_0.[11] When any worker seek their jobs in period 1, the mean and variance of IPU are represented as:

$$\mu_{\rho 1} \equiv \sum_{x_0=0}^{b} \sum_{\Delta x=0}^{b(x_0)} \rho(\Delta x; x_0) f(\Delta x | x_0) f(x_0),$$

$$\sigma_{\rho 1}^2 \equiv \sum_{x_0=0}^{b} \sum_{\Delta x=0}^{b(x_0)} [\rho(\Delta x; x_0) - \mu_{\rho 1}]^2 f(\Delta x | x_0) f(x_0).$$

We can observe the mean-preserving property of IPU between the two periods:

$$\mu_{\rho 1} = \sum_{x_0=0}^{b} \rho(x_0) f(x_0) = \mu_{\rho 0}. \tag{21}$$

In the following lemma, we see the mean and variance of IPU of an economy via those of the two groups, U_0 and E_0.

Lemma 3: *(Decomposition of the IPU distribution in period 1). Suppose Assumption (A1) holds. The mean of IPU of an economy in period 1 can be represented as the weighted average between $\mu_{\rho 1}(U_0)$ and $\mu_{\rho 1}(E_0)$:*

$$\mu_{\rho 1} = l(U_0)\mu_{\rho 1}(U_0) + l(E_0)\mu_{\rho 1}(E_0). \tag{23}$$

The variance of IPU of the economy in period 1 can be decomposed into the weighted average of the variances of U_0 and E_0 and the variance of the economy in period 0:

$$\sigma_{\rho 1}^2 = l(U_0)\sigma_{\rho 1}^2(U_0) + l(E_0)\sigma_{\rho 1}^2(E_0) + (\frac{1}{\mu_{\rho 0}} + \frac{1}{1-\mu_{\rho 0}})\sigma_{\rho 0}^4. \tag{24}$$

Proof: See the Appendix B. ‖

[11] In the Appendix A, by focusing the progress of automation, we examine the dynamics of IPU distribution in an economy.

The mean of IPU distribution of an economy is equal to the weighted average of the two groups, U_0 and E_0. The variance of the economy partly depends on the weighted average of the variances for the two groups. The variance of the economy in period 0 itself increases that in period 1.

In Figure 5, we illustrate the decomposition of the IPU distribution of an economy into those distributions of the groups, U_0 and E_0. The distribution of the group U_0 which is represented as the bold line is given by $f(\Delta x|x_0, U_0)\rho(x_0)f(x_0; \Gamma_0)$. The distribution of the group E_0 which is represented as the dotted line is given by $f(\Delta x|x_0, E_0)(1 - \rho(x_0))f(x_0; \Gamma_0)$. The sum of the two distributions corresponds with the distribution of the economy which is given by $f(x_1, \Gamma_1)$. The difference in the two distributions between the economy and the group E_0 increases with large IPU of the group U_0 because it is more likely to have unemployment.

Proposition 3: *(IPU Distributions of the two groups, U_0 and E_0 in period 1). Suppose Assumption (A1) holds. A large variance of the IPU in the initial period increases the next period's mean of the IPU in the group, U_0, but decreases that in the group, E_0. That is, it can imply the bipolarization in the IPU distributions between the two groups, U_0 and E_0. Furthermore, the variances of IPU in the two groups with a rapid progress in automation.*

4. Concluding remarks

This study sheds light on the IPU of workers that underlie the macroeconomic unemployment rate. Under automation, the variance of the IPU increases with no change in its mean, which implies that some workers have more anxiety concerning their employment, even when the unemployment rate of the economy does not increase. Examining a random matching between jobs and workers as the benchmark, we showed that the job mismatch can be potentially severe because automation reveals the heterogeneity among workers. Furthermore, we showed the possibility of unemployment persistence in the dynamics of IPU.

Our model can be applied to examine unemployment caused by changes in the number of available jobs. It is possible to examine the effects of technological innovations on a decrease in unemployment caused by an increase in the job availability. Furthermore, we can examine how the specialization by trade affects the unemployment associated with the movement of labor.

Appendix A. IPU under the progress in automation

In this appendix, by focusing the progress in automation, we examine the dynamics of IPU distribution in an economy. We consider that not only workers who cannot work in period 0, but also workers who work in that period seek their jobs in period 1.

We first see the expected value and variance of the IPU for the x_0-th group in period 1, respectively:

$$\mu_{\rho 1}(\Delta x|x_0) \equiv E(\rho(\Delta x; x_0)) = \sum_{\Delta x=0}^{b(x_0)} \rho(\Delta x; x_0) f(\Delta x|x_0),$$

$$\sigma^2_{\rho 1}(\Delta x|x_0) \equiv Var(\rho(\Delta x;x_0)) = \sum_{\Delta x=0}^{b(x_0)} [\rho(\Delta x;x_0) - E(\rho(\Delta x;x_0))]^2 f(\Delta x|x_0).$$

Under Assumption (A1), the expected value and variance of the IPU for the x_0-th group in period 1 are, respectively, represented as follows:

$$\mu_{\rho 1}(\Delta x|x_0) = \rho(x_0), \quad \sigma^2_{\rho 1}(\Delta x|x_0) = \sigma^2_{\phi 1}(\Delta x|x_0)v^2,$$

where

$$\sigma^2_{\phi 1}(\Delta x|x_0) = \frac{\Delta\Gamma}{N(\Gamma_0) - \Delta\Gamma}\frac{N(\Gamma_0) - b(x_0)}{N(\Gamma_0) - 1}\frac{b(x_0)}{N(\Gamma_0)^2}.$$

The expected value of IPU for the x_0-th group in period 1 is equal to the IPU in period 0. Automation has no effect on the expected value of IPU in period 1. Thus, when workers have only a small number of unsuited jobs which are covered by machines in period 0, it implies not only a large IPU in period 0, but also a large expected value of the IPU in period 1. Figure A2 illustrates the dynamics of IPU distribution in an economy. In the figure, we define: $x_1 \equiv x_0 + \Delta x$ and $\Gamma_1 \equiv \Gamma + \Delta\Gamma$. The dotted lines represent the distributions of IPU in periods 0 and 1. The IPU distribution evolves while it preserves the mean.

As illustrated in Figure (A1), in period 1, the variance of IPU for the x_0-th group appears because the progress in automation have the different effects on the IPU. In any x_0-th group, the IPU of some workers will surely increase. Furthermore, the variance of the IPU in period 1 can be large when the IPU in period 0 is large. Thus, workers who have a high possibility of unemployment in period 0 can face a more difficulty to obtain employment in period 1. It implies unemployment persistence by the progress of automation, without assuming the correlation of unemployment between the two periods.

We now examine the expected value of IPU of an economy in period 1 from the view point of Δx:

$$\mu_{\rho 1} = \sum_{x_0=0}^{b} \sum_{\Delta x=0}^{b(x_0)} \rho(\Delta x;x_0) f(\Delta x|x_0) f(x_0)$$

$$= \sum_{x_0=0}^{b} \mu_{\rho 1}(\Delta x) f(x_0) = 1 - (1 - \mu_{\phi 0})v = \mu_{\rho 0}. \qquad (B1)$$

$\mu_{\phi 0} = \frac{b}{N_0}$.

The expected value of IPU of an economy remains unchanged even with the progress of automation. On the average, workers who have a large IPU in period 0 will have the difficulty to obtain employment in period 1 while workers who have a small IPU will easily obtain employment in period 1. These cancel each other out in the economy.

We next see the variance of IPU of an economy in period 1:

$$\sigma^2_{\rho 1} = \sum_{x_0=0}^{b} \sum_{\Delta x=0}^{b(x_0)} [\rho(\Delta x;x_0) - \mu_{\rho 1}]^2 f(\Delta x|x_0) f(x_0)$$

$$= \sum_{x_0=0}^{b} \sum_{\Delta x=0}^{b(x_0)} [\rho(\Delta x; x_0) - \mu_\rho(\Delta x|x_0) + \mu_\rho(\Delta x; x_0) - \mu_{\rho 1}]^2 f(\Delta x|x_0) f(x_0).$$

The variance of IPU in period 1 can be represented as:

$$\sigma^2_{\rho 1} = \sum_{x_0=0}^{b} \sigma^2_{\rho 1}(\Delta x|x_0) f(x_0) + \sum_{x_0=0}^{b} \sum_{\Delta x=0}^{b(x_0)} [\mu_\rho(\Delta x|x_0) - \mu_{\rho 1}]^2 f(\Delta x|x_0) f(x_0),$$

because the cross term is zero:

$$\sum_{x=0}^{b} \sum_{\Delta x=0}^{b(x_0)} [\rho(\Delta x; x_0) - \mu_\rho(\Delta x|x_0)][\mu_\rho(\Delta x|x_0) - \mu_{\rho 1}] f(\Delta x|x_0) f(x_0) = 0.$$

We have:

$$\sum_{x_0=0}^{b} \sum_{\Delta x=0}^{b(x_0)} [\mu_\rho(\Delta x|x_0) - \mu_{\rho 1}]^2 f(\Delta x|x_0) f(x_0) = \sum_{x_0=0}^{b} [\rho(x_0) - \mu_{\rho 1}]^2 f(x_0) = \sigma^2_{\rho 0}.$$

Consequently, under Assumption (A1), we obtain:

$$\sigma^2_{\rho 1} = \sum_{x_0=0}^{b} \sigma^2_{\rho 1}(\Delta x|x_0) f(x_0) + \sigma^2_{\rho 0}. \qquad (B2)$$

Any x_0-th group has the own variance in period 1. Thus, the variance of IPU in the economy increases because of $\Delta\Gamma > 0$, that is, $\sigma^2_{\rho 1} > \sigma^2_{\rho 0}$. The possibility of mismatch unemployment in period 0 can increase for workers who have high IPU in period 0.

Appendix B. Proof of Lemma 3

We decompose $\sigma^2_{\rho 1}$ into the two parts which correspond with the two groups, U_0 and E_0:

$$\sigma^2_{\rho 1} = \sigma^2_{\rho 1}|_{U_0} + \sigma^2_{\rho_1}|_{E_0}, \qquad (B3)$$

where

$$\sigma^2_{\rho 1}|_{U_0} \equiv \sum_{x_0=0}^{b} \sum_{\Delta x=0}^{b(x_0)} [\rho(\Delta x; x_0) - \mu_{\rho 1}]^2 f(\Delta x|x_0) \rho(x_0) f(x_0),$$

$$\sigma^2_{\rho 1}|_{E_0} \equiv \sum_{x_0=0}^{b} \sum_{\Delta x=0}^{b(x_0)} [\rho(\Delta x; x_0) - \mu_{\rho 1}]^2 f(\Delta x|x_0)(1 - \rho(x_0)) f(x_0).$$

The first term in (B3) which corresponds with the group U_0 can be represented as:

$$\sigma^2_{\rho 1}|_{U_0} = \sum_{x_0=0}^{b} \sum_{\Delta x=0}^{b(x_0)} [\rho(\Delta x; x_0, U_0) - \mu_{\rho 1}(U_0) + \mu_{\rho 1}(U_0) - \mu_{\rho 1}]^2 f(\Delta x|x_0) \rho(x_0) f(x_0)$$

$$= \sum_{x_0=0}^{b} \sum_{\Delta x=0}^{b(x_0)} \{[\rho(\Delta x; x_0, U_0) - \mu_{\rho 1}(U_0)]^2 + [\mu_{\rho 1}(U_0) - \mu_{\rho 1}]^2\} f(\Delta x|x_0)\rho(x_0)f(x_0),$$

because the cross term is equal to zero:

$$\sum_{x_0=0}^{b} \sum_{\Delta x=0}^{b(x_0)} [\rho(\Delta x; x_0, U_0) - \mu_{\rho 1}(U_0)][\mu_{\rho 1}(U_0) - \mu_{\rho 1}] f(\Delta x|x_0)\rho(x_0)f(x_0) = 0.$$

By use of the definition of $\sigma^2_{\rho 1}(U_0)$, we obtain:

$$\sigma^2_{\rho 1}|_{U_0} = l(U_0)\sigma^2_{\rho 1}(U_0) + (\frac{\sigma^2_{\rho 0}}{\mu_{\rho 0}})^2 \mu_{\rho 0},$$

where

$$\sigma^2_{\rho 1}(U_0) \equiv \frac{1}{l(U_0)} \sum_{x_0=0}^{b} \sum_{\Delta x=0}^{b(x_0)} [\rho(\Delta x; x_0, U_0) - \mu_{\rho 1}(U_0)]^2 f(\Delta x|x_0)\rho(x_0)f(x_0).$$

Similarly, by use of the definition of $\sigma^2_{\rho 1}(E_0)$, the second term in (B3) which corresponds with the group E_0 can be represented as:

$$\sigma^2_{\rho 1}|_{E_0} = l(E_0)\sigma^2_{\rho 1}(E_0) + (\frac{\sigma^2_{\rho 0}}{1 - \mu_{\rho 0}})^2 (1 - \mu_{\rho 0}),$$

where

$$\sigma^2_{\rho 1}(E_0) \equiv \frac{1}{l(E_0)} \sum_{x_0=0}^{b} \sum_{\Delta x=0}^{b(x_0)} [\rho(\Delta x; x_0, E_0) - \mu_{\rho 1}(E_0)]^2 f(\Delta x|x_0)\rho(x_0)f(x_0).$$

Consequently, we can represent the variance of IPU of an economy in period 1 as follows:

$$\sigma^2_{\rho 1} = l(U_0)[\sigma^2_{\rho 1}(U_0) + (\frac{\sigma^2_{\rho 0}}{\mu_{\rho 0}})^2] + l(E_0)[\sigma^2_{\rho 1}(E_0) + (\frac{\sigma^2_{\rho 0}}{1 - \mu_{\rho 0}})^2].$$

Appendix C. Two types of job categories and two types of workers

In this appendix, we examine an augmented model with two types of job categories: one type requires physical strength while the second type requires intelligence. We also extend out model by considering two types of individuals.

We divide the Nth number of job categories into two types which are physical and intelligent jobs:

$$N = N_P + N_I,$$

where N is the total number of job categories. N_P and N_I are respectively, the numbers of physical and intelligent job categories.

We represent the number of job categories that are automated:

$$\Gamma = \Gamma_P + \Gamma_I,$$

where Γ is the total number of automated jobs. Γ_P and Γ_I are respectively, the numbers of physical and intelligent jobs that are automated.

We define the ratio of physical jobs that are not automated: $\theta_P \equiv \frac{N_P - \Gamma_P}{N-\Gamma}$. Thus, the ratio of intelligent jobs that are not automated is represented as: $1-\theta_P = \frac{N_I-\Gamma_I}{N-\Gamma}$

Individuals randomly have their suitable and unsuitable jobs both in the physical jobs and in the intelligent jobs. We represent the numbers of unsuited jobs that machines cover as:

$$x = x_P + x_I,$$

where x is the total number of unsuited jobs that machines cover. x_P and x_I represent those numbers of physical and intelligent jobs, respectively.

We examine individual mismatch rates:

$$\phi(x) = \frac{b-x}{N-\Gamma} = \frac{N_P-\Gamma_P}{N-\Gamma}\frac{b_P-x_P}{N_P-\Gamma_P} + \frac{N_I-\Gamma_I}{N-\Gamma}\frac{b_I-x_I}{N_I-\Gamma_I}$$
$$= \theta_P\phi(x_P) + (1-\theta_P)\phi(x_I),$$

where $\phi(x_P) = \frac{b_P-x_P}{N_P-\Gamma_P}$ and $\phi(x_I) = \frac{b_I-x_I}{N_I-\Gamma_I}$.

The mean of mismatch rates is represented as follows:

$$\mu_\phi \equiv \theta_P \sum_{x_P=0}^{b_P} \phi(x_P)f(x_P) + (1-\theta_P)\sum_{x_I=0}^{b_I}\phi(x_I f(x_I),$$

where $f(x_P)$ and $f(x_I)$ are respectively, the following hyper geometric distributions:

$$f(x_P) \equiv \frac{\begin{pmatrix}\Gamma_P \\ x_P\end{pmatrix}\begin{pmatrix}N_P-\Gamma_P \\ b_P-x_P\end{pmatrix}}{\begin{pmatrix}N_P \\ b_P\end{pmatrix}} \quad and \quad f(x_I) \equiv \frac{\begin{pmatrix}\Gamma_I \\ x_I\end{pmatrix}\begin{pmatrix}N_I-\Gamma_I \\ b_I-x_I\end{pmatrix}}{\begin{pmatrix}N_I \\ b_I\end{pmatrix}}.$$

Thus, we obtain:

$$\mu_\phi = \theta_P\frac{b_P}{N_P} + (1-\theta_P)\frac{b_I}{N_I}. \qquad (B4)$$

For simplicity, by assuming that $v = 1$, we consider the IPU which is equal to the individual mismatch rates. We examine the effect of automation for physical jobs on the mean of individual mismatch rates:

$$\frac{\partial\mu_\phi}{\partial\Gamma_P} = \frac{\partial\theta_P}{\partial\Gamma_P}\frac{b_P}{N_P} + \frac{\partial(1-\theta_P)}{\partial\Gamma_P}\frac{b_I}{N_I}$$
$$= \frac{N_I-\Gamma_I}{(N-\Gamma)^2}\left(-\frac{b_P}{N_P} + \frac{b_I}{N_I}\right).$$

Furthermore, the effect of automation for intelligent jobs is represented as:

$$\frac{\partial \mu_\phi}{\partial \Gamma_I} = \frac{\partial \theta_P}{\partial \Gamma_I}\frac{b_P}{N_P} + \frac{\partial (1-\theta_P)}{\partial \Gamma_I}\frac{b_I}{N_I}$$
$$= \frac{N_P - \Gamma_P}{(N-\Gamma)^2}(\frac{b_P}{N_P} - \frac{b_I}{N_I}).$$

Automation can change the mean of individual mismatch rates via the change in the weights of two job categories when we have the difference in the relative suitability between the physical jobs and intelligent jobs:

$$\frac{b_P}{N_P} \underset{<}{>} \frac{b_I}{N_I}. \qquad (B5)$$

Depending on the inequality in (B5), automation of physical and intelligent jobs have the different effects on the macroeconomic unemployment rate. Thus, the macroeconomic unemployment rate would change as a result of automation because of the difference in the relative suitability of jobs.

We also extend our model by considering two types of individuals, α and β. The size of total population is represented as: $L = L(\alpha) + L(\beta)$ in which $L(\cdot)$ represents the population size of each type. We define the population ratio of type α as $\lambda(\alpha) \equiv \frac{L(\alpha)}{L}$. Thus, the population ratio of type β is equal to $1 - \lambda(\alpha)$.

Either the two types of individuals randomly have their suitable and unsuitable jobs both in the physical jobs and in the intelligent jobs. Individuals of each type are assumed to have the same numbers of unsuited jobs for physical and intelligent jobs:

$$b(\alpha) = b_P(\alpha) + b_I(\alpha) \;\; and \;\; b(\beta) = b_P(\beta) + b_I(\beta),$$

where $b(\cdot)$ represents the total number of unsuited jobs. $b_P(\cdot)$ and $b_I(\cdot)$ represent the numbers of unsuited physical and unsuited intelligent jobs, respectively.

We assume that individuals of type α are relatively more suitable for physical jobs than for intelligent jobs:

$$\frac{b_P(\alpha)}{N_P} < \frac{b_I(\alpha)}{N_I}. \qquad (B6)$$

Furthermore, we assume that individuals of type β are relatively more suitable for intelligent jobs than for physical jobs:

$$\frac{b_P(\beta)}{N_P} > \frac{b_I(\beta)}{N_I}. \qquad (B7)$$

We first see the individual mismatch rates of type α:

$$\phi(x;\alpha) = \frac{b(\alpha) - x}{N - \Gamma} = \theta_P \phi(x_P;\alpha) + (1-\theta_P)\phi(x_I;\alpha),$$

where $\phi(x_P;\alpha) = \frac{b_P(\alpha) - x_P}{N_P - \Gamma_P}$ and $\phi(x_I;\alpha) = \frac{b_I(\alpha) - x_I}{N_I - \Gamma_I}$. Note that $x_P = 0, 1, \cdots, b_P(\alpha)$ and $x_I = 0, 1, \cdots, b_I(\alpha)$.

The mean of individual mismatch rates for the type α which is equal to their IPU mean is represented as:

$$\mu_\phi(\alpha) \equiv \theta_P \sum_{x_P=0}^{b_P(\alpha)} \phi(x_P;\alpha) f(x_P) + (1-\theta_P)) \sum_{x_I=0}^{b_I(\alpha)} \phi(x_I;\alpha) f(x_I)$$

$$= \theta_P \frac{b_P(\alpha)}{N_P} + (1-\theta_P) \frac{b_I(\alpha)}{N_I}. \qquad (B8)$$

Under Assumption (B6), automation of physical jobs increases the IPU mean whereas automation of intelligent jobs decreases the IPU mean:

$$\frac{\partial \mu_\phi(\alpha)}{\partial \Gamma_P} = \frac{N_I - \Gamma_I}{(N-\Gamma)^2}\left(-\frac{b_P(\alpha)}{N_P} + \frac{b_I(\alpha)}{N_I}\right) > 0,$$

$$\frac{\partial \mu_\phi(\alpha)}{\partial \Gamma_I} = \frac{N_P - \Gamma_P}{(N-\Gamma)^2}\left(\frac{b_P(\alpha)}{N_P} - \frac{b_I(\alpha)}{N_I}\right) < 0.$$

We next see the individual mismatch rates of type β:

$$\phi(x;\beta) = \frac{b(\beta) - x}{N - \Gamma} = \theta_P \phi(x_P;\beta) + (1-\theta_P)\phi(x_I;\beta),$$

where $\phi(x_P;\beta) = \frac{b_P(\beta) - x_P}{N_P - \Gamma_P}$ and $\phi(x_I;\beta) = \frac{b_I(\beta) - x_I}{N_I - \Gamma_I}$. Note that $x_P = 0, 1, \cdots, b_P(\beta)$ and $x_I = 0, 1, \cdots, b_I(\beta)$.

The mean of individual mismatch rates for the type β which is equal to their IPU mean is represented as:

$$\mu_\phi(\beta) \equiv \theta_P \sum_{x_P=0}^{b_P(\beta)} \phi(x_P;\beta) f(x_P) + (1-\theta_P)) \sum_{x_I=0}^{b_I(\beta)} \phi(x_I;\beta) f(x_I)$$

$$= \theta_P \frac{b_P(\beta)}{N_P} + (1-\theta_P) \frac{b_I(\beta)}{N_I}. \qquad (B9)$$

Under Assumption (B7), automation of physical jobs decreases the IPU mean whereas automation of intelligent jobs increases the IPU mean:

$$\frac{\partial \mu_\phi(\beta)}{\partial \Gamma_P} = \frac{N_I - \Gamma_I}{(N-\Gamma)^2}\left(-\frac{b_P(\beta)}{N_P} + \frac{b_I(\beta)}{N_I}\right) < 0,$$

$$\frac{\partial \mu_\phi(\beta)}{\partial \Gamma_I} = \frac{N_P - \Gamma_P}{(N-\Gamma)^2}\left(\frac{b_P(\beta)}{N_P} - \frac{b_I(\beta)}{N_I}\right) > 0.$$

Finally, we examine the IPU mean of an economy:

$$\mu_\phi \equiv \lambda(\alpha)\mu_\phi(\alpha) + (1-\lambda(\alpha))\mu_\phi(\beta). \qquad (B10)$$

The effect of automation of physical jobs on the IPU mean depends on the population ratios and relative job suitability of the two types, α and β:

$$\frac{\partial \mu_\phi}{\partial \Gamma_P} = \lambda(\alpha)\frac{\partial \mu_\phi(\alpha)}{\partial \Gamma_P} + (1-\lambda(\alpha))\frac{\partial \mu_\phi(\beta)}{\partial \Gamma_P}$$

$$= \frac{N_I - \Gamma_I}{(N-\Gamma)^2}[\lambda(\alpha)(-\frac{b_P(\alpha)}{N_P} + \frac{b_I(\alpha)}{N_I}) + (1-\lambda(\alpha))(-\frac{b_P(\beta)}{N_P} + \frac{b_I(\beta)}{N_I})].$$

Furthermore, the effect of automation of intelligent jobs on the IPU mean is represented as:

$$\frac{\partial \mu_\phi}{\partial \Gamma_I} = \lambda(\alpha)\frac{\partial \mu_\phi(\alpha)}{\partial \Gamma_I} + (1-\lambda(\alpha))\frac{\partial \mu_\phi(\beta)}{\partial \Gamma_I}$$

$$= \frac{N_P - \Gamma_P}{(N-\Gamma)^2}[\lambda(\alpha)(\frac{b_P(\alpha)}{N_P} - \frac{b_I(\alpha)}{N_I}) + (1-\lambda(\alpha))(\frac{b_P(\beta)}{N_P} - \frac{b_I(\beta)}{N_I})].$$

Thus, the effects of automation for physical and intelligent jobs depend on the relative suitability of jobs between two types of workers.

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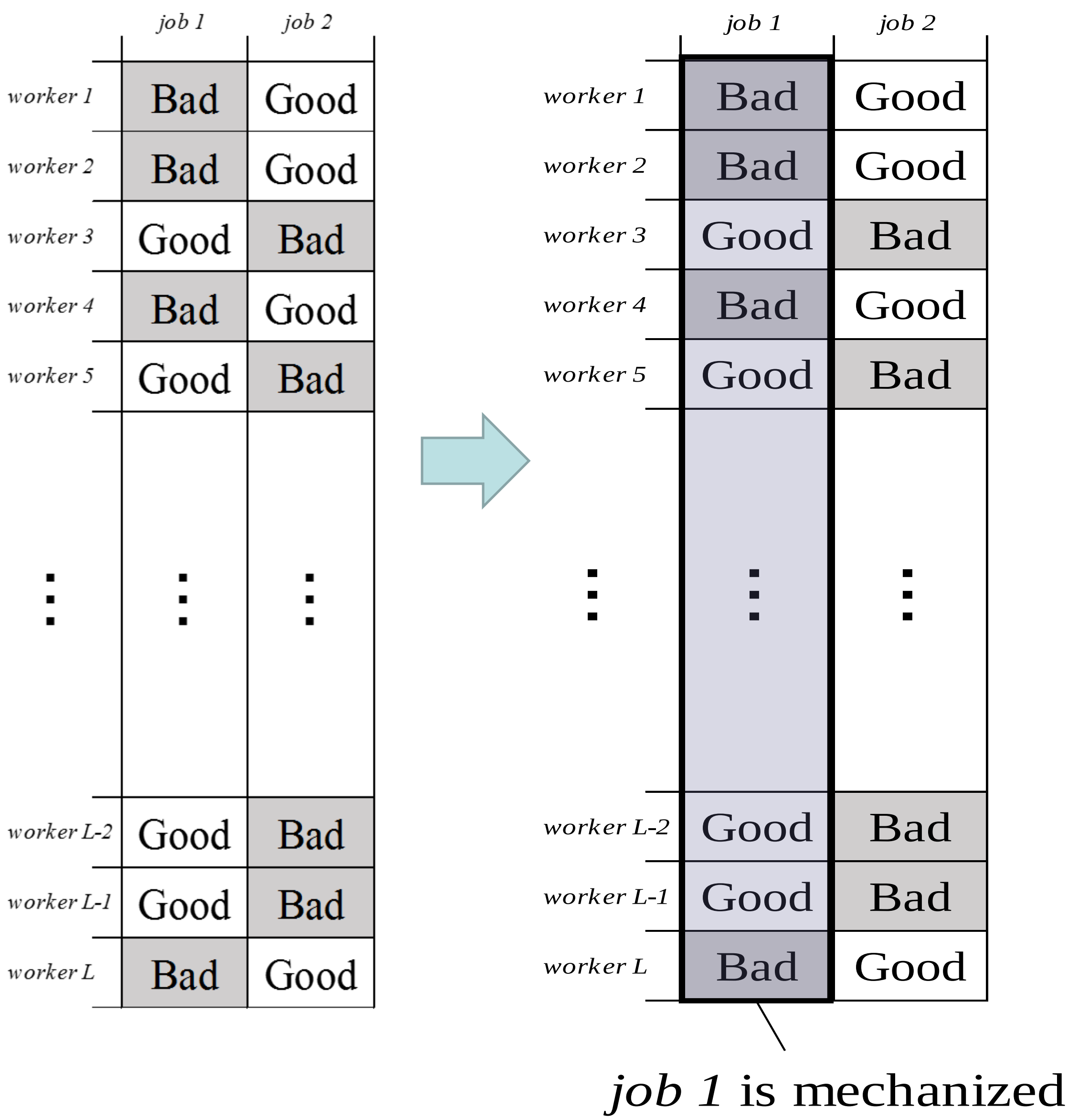

Figure 1. Suitable and unsuitable job categories of workers: An example

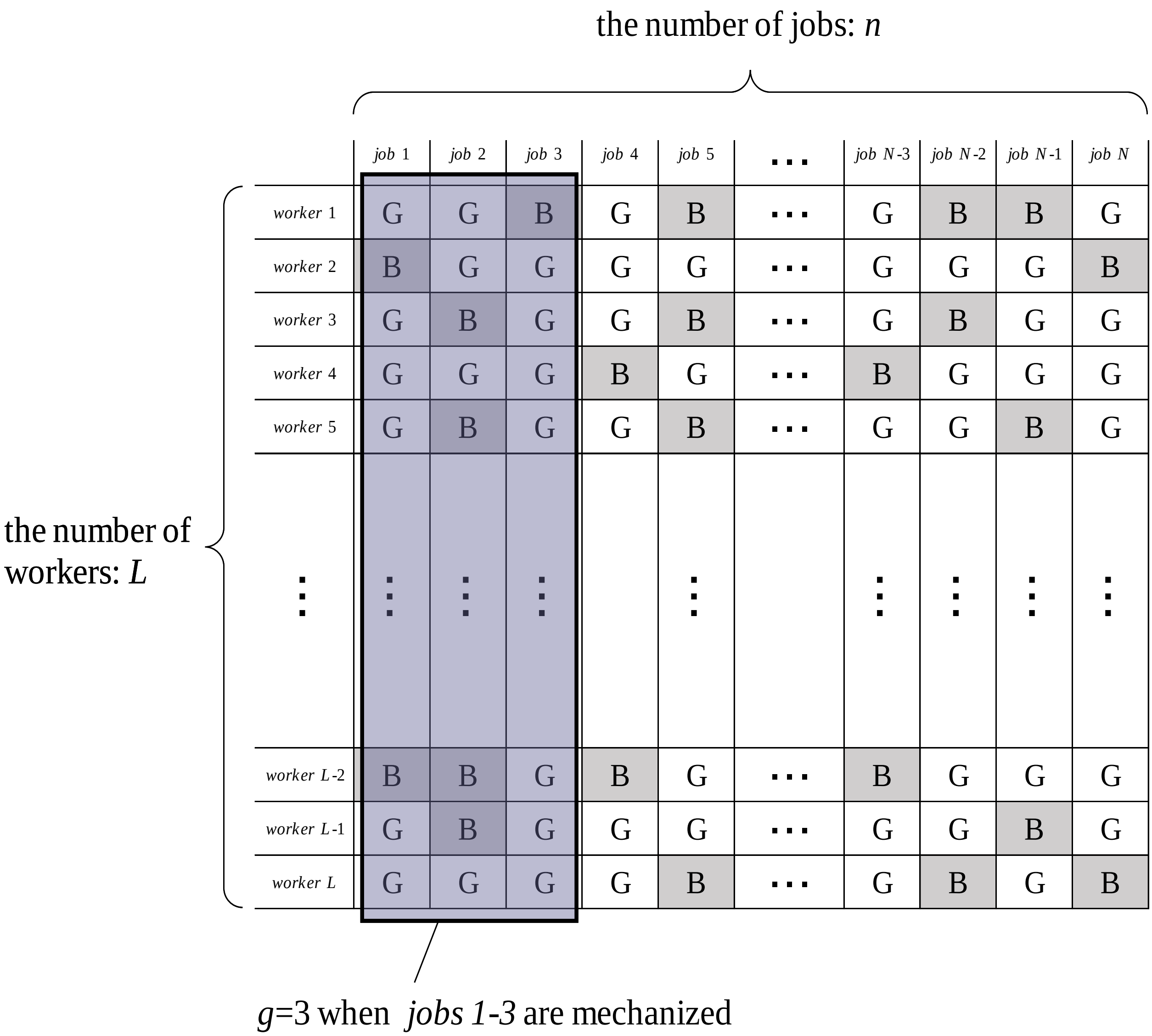

Figure 2. Suitable and unsuitable job categories of workers: A general model

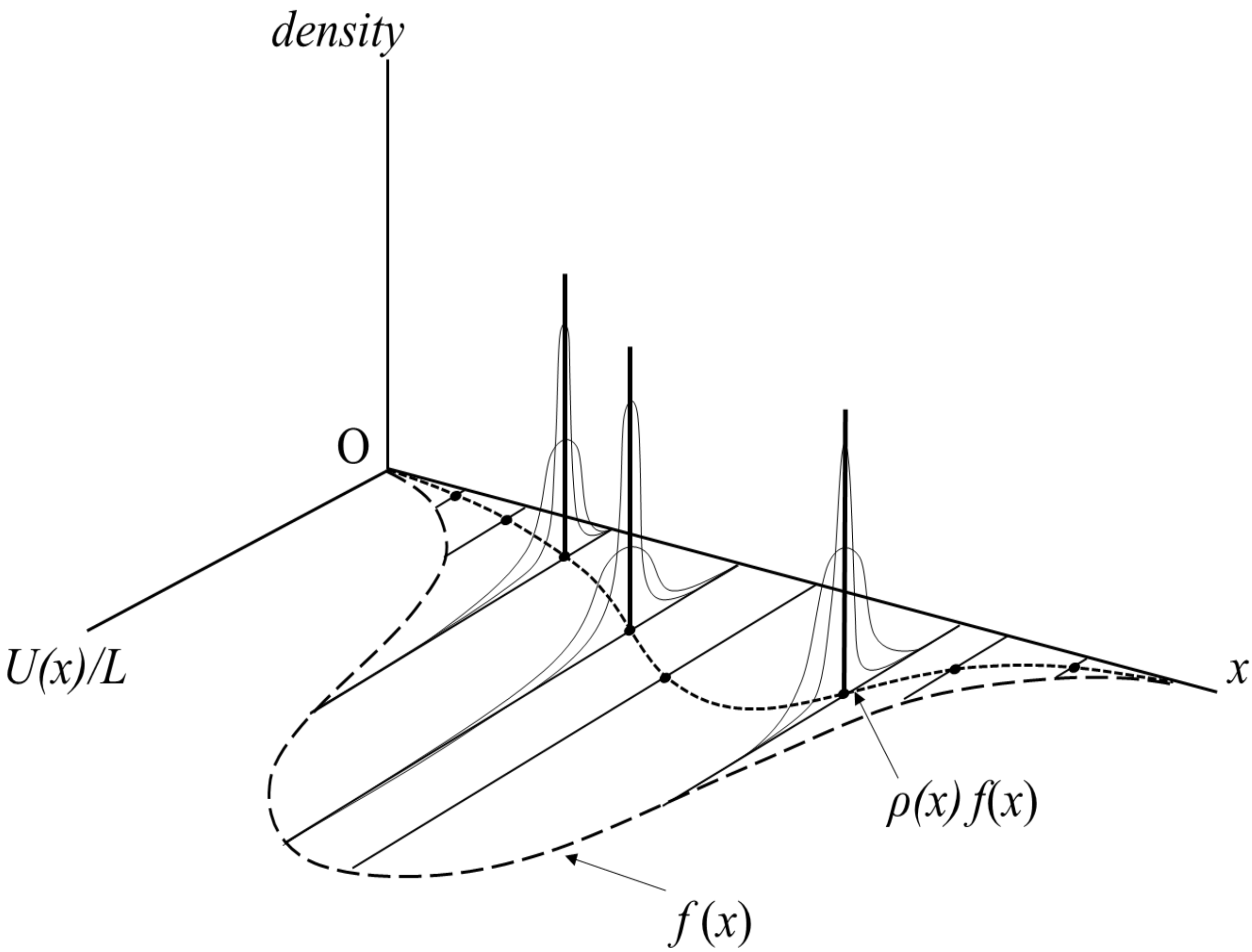

Figure 3. Macroeconomic unemployment rate via IPU

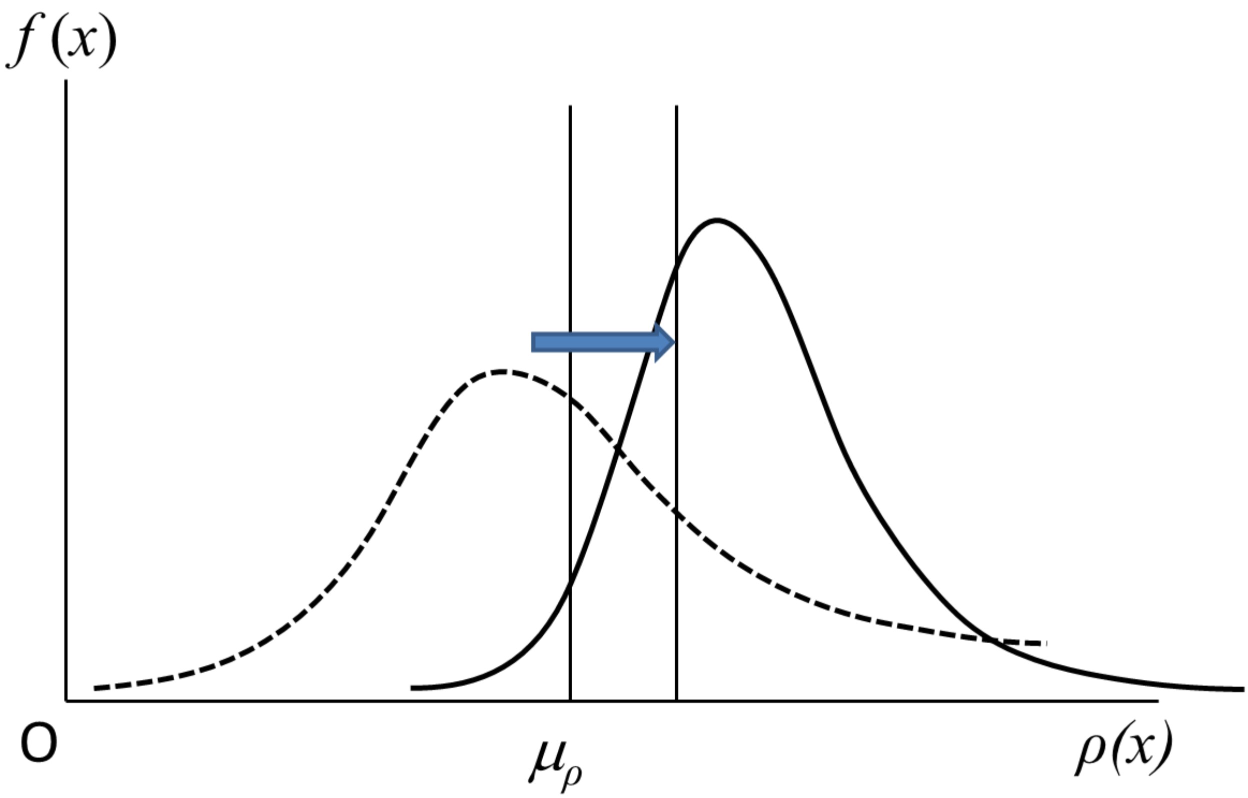

Figure 4. Distribution of IPU

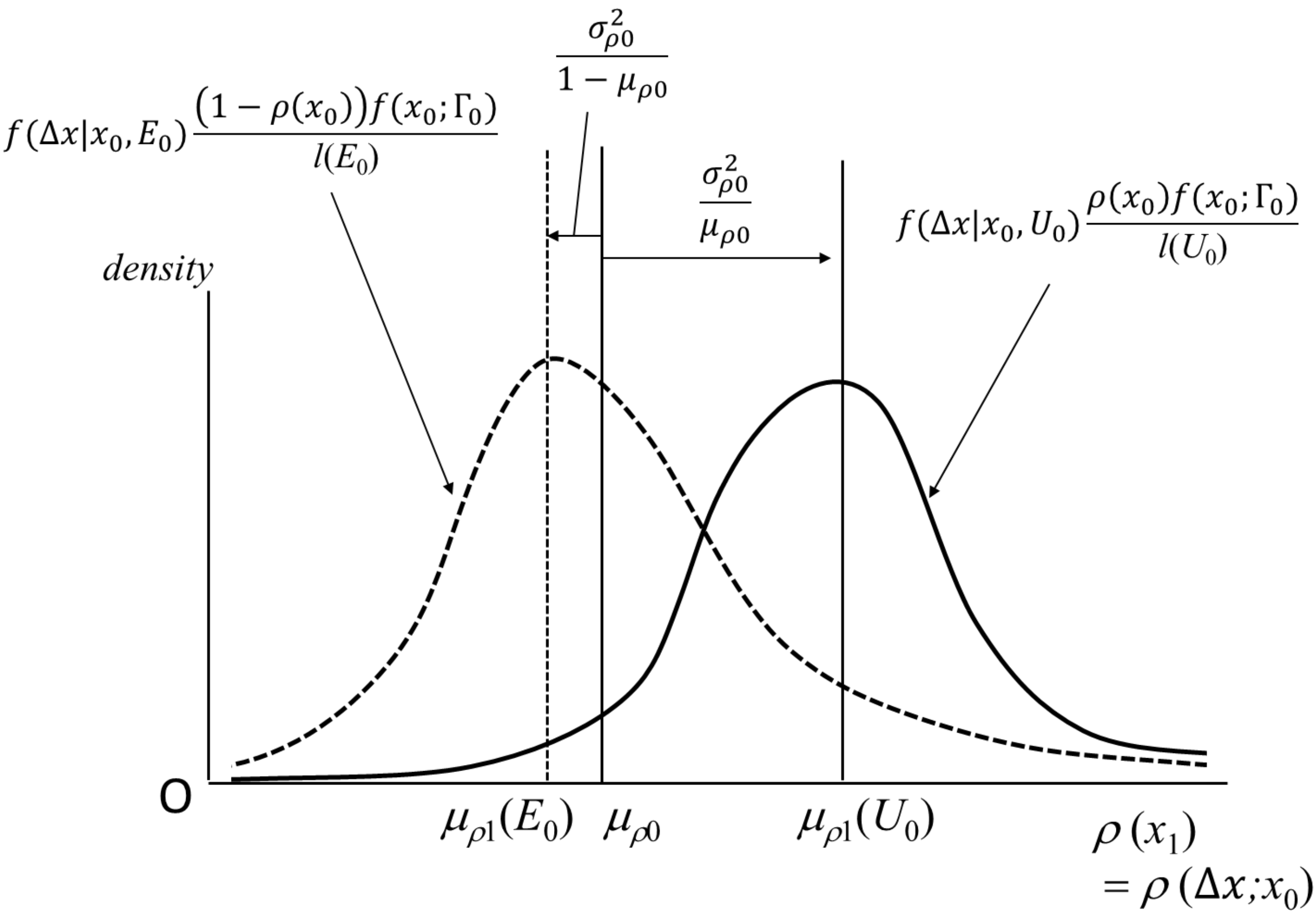

Figure 5. IPU distributions of the two groups, U_0 and E_0 in period *1*

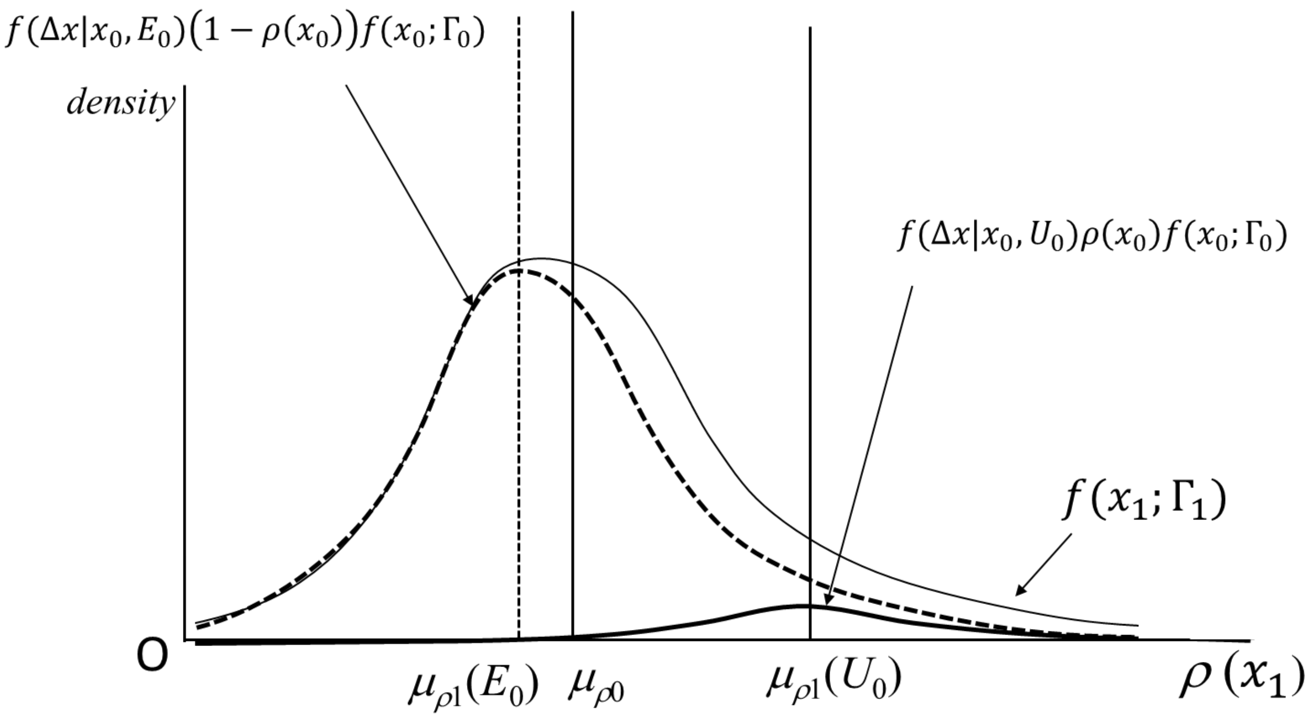

Figure 6. Decomposition of IPU distribution into the two groups, U_0 and E_0

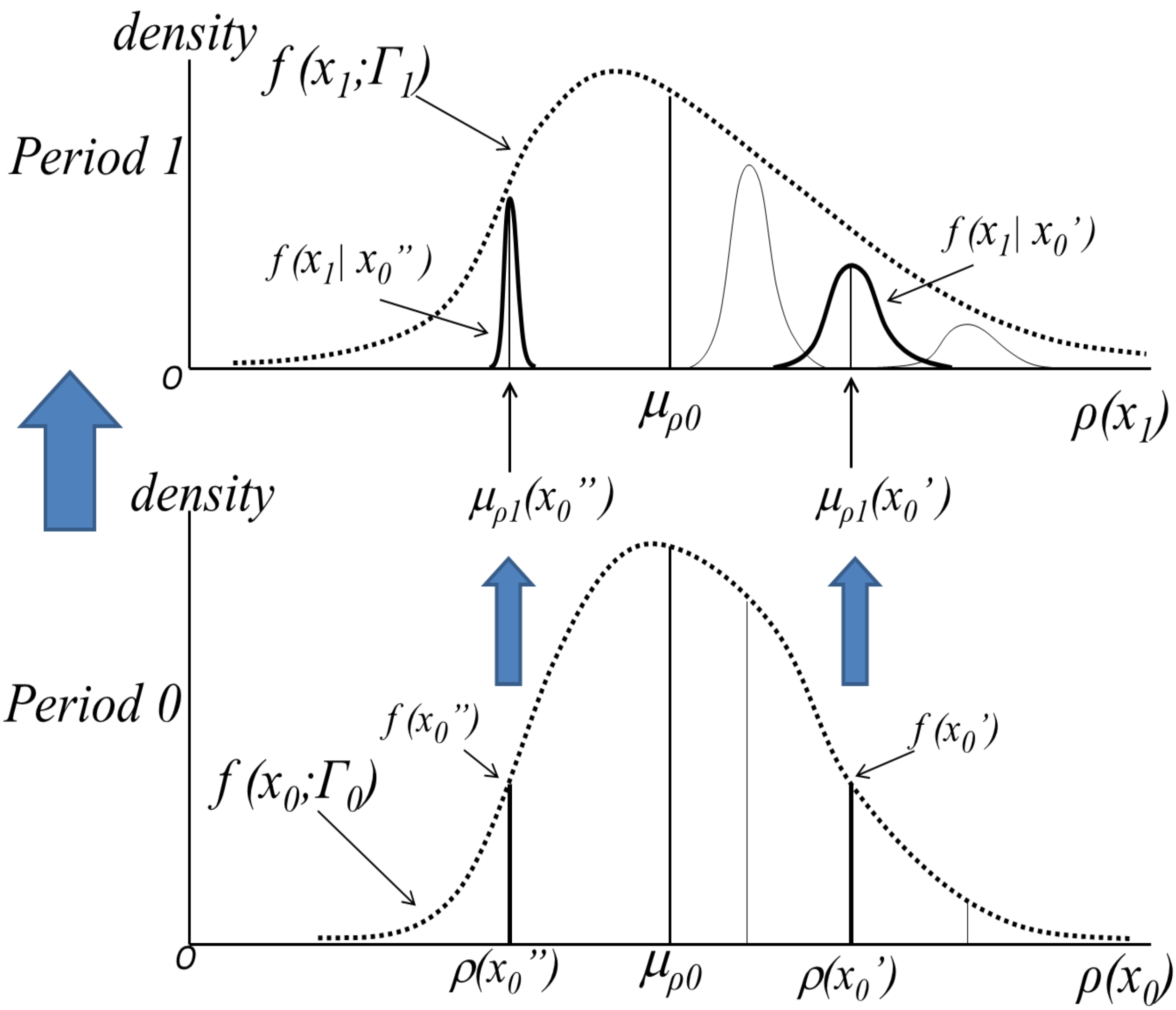

Figure A1. Dynamics of IPU in an economy