# The Housing Cost Disease *

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**Abstract**

Using a simple two-sector life cycle economy with bequests, we show that a rising labor efficiency in the general economy relative to the construction sector can go a long way toward explaining a significant fraction of the rising trends in wealth-to-income ratios, housing wealth, and wealth inequality, that have been documented in most advanced countries at least since the '70s. When consumption inequality across households is sufficiently large, this mechanism (which we label *housing cost disease*) has adverse effects on a measure of social welfare based on an egalitarian principle: the higher the housing's value appreciation, the lower the welfare benefit of a rising relative labor efficiency.

KEYWORDS: Housing Wealth, Cost Disease, Overlapping Generations, Wealth Inequality.

JEL CODES: D91, O11, H2, G1.

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# 1 Introduction

Wealth-to-income ratios have been increasing in most advanced economies at least since the 1970s, and housing wealth, which accounts for about 35% of the total, has been the main driver in many countries. Because wealth is more unevenly distributed than income, a concern for a widening inequality gap is emerging in academic and non-academic circles. In fact, in advanced countries, wealth inequality, while declining in the period 1950 to 1970, has been increasing at least since 1980.

A growing literature suggests that, to understand the existing trends in wealth inequality, wealth ratios and income shares, it is crucial to analyze the dynamics of housing, since much of the long-term dynamics of the capital-income ratio, as well as the net capital share of income, is accounted for by housing wealth and, in particular, by capital gains (for example, Bonnet et al. (2014), Piketty and Zucman (2014), Rognlie (2014) and Summers (2014)). Following this literature, we provide an explanation of these long run trends based on a sort of Baumol's cost disease. In particular, we build a frictionless two-sector life-cycle model with bequests able to replicate some of the stylized facts concerning the dynamics of total wealth, housing wealth, and wealth inequality, as a consequence of an improvement in the efficiency of labor in the rest of the economy relative to the construction sector.

In the seminal work by Baumol (1967), a market economy has two sectors producing two goods using labor as the only input and enjoying different patterns of technological progress. Under perfect labor mobility and wage equalization, a rising labor productivity in the dynamic sector generates a higher production cost and, then, a rising relative output price in the stagnant sector. If the demand of the stagnant sector output is sufficiently inelastic, labor will move to this sector and aggregate output growth may decline. We extend Baumol's analysis to a life-cycle model with bequests (generated by parental altruism). The two sectors are called *construction* and *manufacturing*, where the former should be interpreted as production of buildings and constructions and the latter as capturing the non-construction sectors of the economy. Technology employs capital and land, as well as labor, and construction plays the role of the stagnant sector, while manufacturing experiences labor-augmenting technological progress. Assuming an altruistic bequest motive, we define the conditions for a rise in the efficiency of labor in manufacturing to generate a strong housing price appreciation, a rise in the wealth-to-income ratio (mostly driven by higher housing and land appreciations and a weak dynamic in average labor productivity) and in the size of bequests (*i.e.*, a rising wealth inequality). We refer to this set of results as *the housing cost disease*. Data for the 8 largest advanced economies over the period 1970-2010 confirm the existence of a positive and large correlation between a labor-augmenting productivity residual in the rest of the economy, relative to the construction sector, and the total and housing wealth-to-income ratios. A multi-sector approach is important to study the evolution of the wealth composition in advanced economies (vs. economies in the early stage of the development process) since housing replaced land in households' assets and because factor price equalization across sectors generates interesting linkages between the dynamics of productivity and asset prices.

We solve the model analytically under the assumption of Cobb-Douglas production functions and show that the housing cost disease is most likely when manufacturing is more capital intensive than construction, the land share of income is not too large, and housing demand is sufficiently inelastic with respect to its own price. This elasticity plays an important role. In particular, with unitary elasticity of substitution between capital and labor in construction, and between goods (consumption and housing services), in a CES representation of preferences, a rising productivity in manufacturing is allocation neutral, in the sense that total and housing wealth-to-income ratios, as well as the shares of labor across sectors, remain unchanged, whereas bequests (of rich households) increase on a one-to-one basis. When, instead, housing demand is sufficiently inelastic, the housing cost disease holds and bequests, and thus inequality, increase more than proportionally with relative productivity in manufacturing. The robustness of these results is studied numerically for a CES specification of preferences and technology. We show that, for a 75% increase in the (exogenous) relative labor efficiency in manufacturing (a value that we consider a good estimation of the actual average improvements between 1970 and 2007 for the 8 largest advanced economies), total wealth-to-income increases by about 40 percent and housing wealth by approximately 60 percent. These patterns are driven by a strong increase in the housing price (about 50 percent) and by an even stronger increase in the land price. Since the interest rate is greater than the population growth rate at equilibrium, steady state net bequests (*i.e.*, the difference between the present value of bequests received from the previous generation and left to the next one) are a positive component of the rich households' present value of income. Then, assuming that housing is a normal good, bequests and housing wealth are strongly correlated, and this dependence is stronger the higher the equilibrium level of the interest rate. In fact, bequests respond approximately proportionally to an increase in relative labor efficiency. While the large increase in housing and land prices is crucial for our numerical results, these are not driven by an implausibly large increase in the share of workers in the construction sector which ranges between 4 and 6 percent. We should stress, though, that, as the model is very stylized, these numerical exercises cannot be interpreted as a proper quantitative exercise, but rather indicative that the proposed channel could be responsible for some share of the changes observed in the data.

We are not the first to highlight the importance of disaggregated productivity improvements, and changes in the allocation of inputs across sectors, to explain movements in housing prices and wealth. Some notable examples are Davis and Heathcote (2005), Kahn (2008), Iacoviello and Neri (2010), Moro and Nuno (2012), and, more recently, Favilukis et al. (2015) and Grossmann and Steger (2016). In particular, Moro and Nuno provide some evidence that a fall in the relative TFP in the construction sector may be responsible for a surge in housing prices, and Grossmann and Steger highlight the scarcity of land as a contributing factor for the increase in wealth-to-income ratios. Most of the other contributions focus on business cycle analysis and credit frictions.

Our theory complements, rather than replaces, alternative explanations of these stylized facts based on financial constraints (for example, Favilukis et al. (2015) and Iacoviello (2005)), regulation affecting the supply of housing and land (for example, Hsieh and Moretti (2015)), precautionary saving (for example, Gourinchas and Parker (2002) and Castaneda et al. (2003)), as well as the saving induced growth mechanism advocated by Piketty and Zucman (2014) who attribute the rising wealth-to-income ratios to the falling income growth rates and the long-run stability of the saving rate[1]. Piketty and Saez (2014) claim that these trends are responsible for the rising income and wealth inequality. Bonnet et al. (2014), Rognlie (2014) and Weil (2015) argue that the existing trends in wealth-to-income ratios and income shares are strongly determined by the dynamic of housing wealth and capital gains. Other contributions explain the observed stylized facts with a combination of alternative assumptions, such as heterogeneous bequest motives, rate of returns shocks and incomplete markets (as in Krusell and Smith (1998), Hendricks (2007), Benhabib et al. (2011), Benhabib et al. (2015)), or endogenous rates of return due to entrepreneurship or human capital (as in Galor and Zeira (1993), Quadrini (2000), Cagetti and De Nardi (2006) and many others). Our model borrows some of the ingredients employed in this literature, within a drastically simplified framework, to explain the rising wealth inequality that goes along with a change in the sectoral composition of GDP. In particular, as in Favilukis et al. (2015), we focus on the role of bequests in generating wealth inequality by assuming that altruistic finitely-lived households have fixed heterogeneous discount rates and that parents cannot force gifts on their children (one-sided altruism)[2]. This het-

[1]See, in particular, the detailed survey by De Nardi and Fang (2015).

[2]The role of bequests in generating wealth inequality can be hardly ignored. For the US, Gale

erogeneity generates a partition of the set of households at steady states into a subset of *rich* individuals receiving bequests from their parents and a subset of *poor* individuals receiving (and giving) no bequests. Although the assumption of heterogeneous bequest motives across individuals as a driver of wealth inequality is not new and it has been widely exploited in the existing literature, we supplement this theory by suggesting that the Baumol cost disease may amplify bequests-induced inequality through TFP improvements. In particular, our model shows that, when housing demand is sufficiently inelastic with respect to its own price, private wealth, housing prices and bequests are complementary. Then, in this case, a rising TFP in the general economy (relative to the construction sector) has a positive effect on wealth, housing and land prices and on the size of bequests, so that wealth inequality increases.

Piketty (2014) advocates the institution of a wealth tax on the assumption that the increase in wealth-to-income is not desirable because of the implications in terms of unequal distribution of wealth across households. However, if this phenomenon is mostly a consequence of rising housing prices, then policies targeting specifically the housing sector are probably more appropriate, as noted for example by Auerbach and Hasset (2015)[3]. We leave for future research the evaluation of such policies, and use, instead, the model to see if a change in the composition of wealth toward housing, following a rise of efficiency in manufacturing, is desirable in terms of an egalitarian welfare criterion. Deaton and Laroque (2001) investigate the welfare effects of development in a multi-assets life-cycle model and found that the presence of a market for housing determines a portfolio reallocation away from capital towards housing, causing the accumulation of capital to fall short of the *Golden Rule* level. Our welfare criterion is based on an egalitarian welfare function that takes into account the households

and Scholz (1994) find that intergenerational transfers account for at least 50-60% of total wealth accumulation.

[3]Note that housing taxation is, in any case, very controversial, since housing is a consumption good, as well as an asset, and home ownership is much more evenly distributed across individuals than stocks and other financial assets.

heterogeneous degrees of altruism with respect to the next generations and allows for unrestricted transfers across generations. We conclude that, when housing appreciation is sufficiently strong, consumption inequality large and wages low relative to the value of the housing stock, the steady state welfare benefit of a rising labor efficiency in manufacturing can be social welfare diminishing. In principle, a housing appreciation has two opposite effects on welfare. First, it raises the wealth of the poor old households so as to relax the non-negativity constraint on bequest values. Second, it makes housing less affordable. The last effect appears to be stronger than the former and detrimental to an egalitarian social welfare measure when poor households' consumption is too low.

The remainder of the paper is organized as follows. Section 2 present some stylized facts on wealth ratios and inequality for eight advanced economies and a preliminary empirical test of the housing cost disease. Section 3 introduces the model and characterizes steady states with bequests. Section 4 shows the effects of improvements in relative labor productivity and the conditions for a housing cost disease. Section 5 discusses the welfare implications. Section 6 concludes.

## 2 The Data

In this section, we combine the empirical evidence from different sets of stylized facts supporting the *housing cost disease*. First, we look at the evolution of national wealth and of one of its main components, housing wealth, using data from Piketty and Zucman (2014). Second, we look at the evolution of wealth and income inequality, using data from Alvaredo et al. (2016)'s Top World Income Database. Third, we present a simplified model of the housing cost disease to interpret the data. Fourth, we show the existence of a positive relationship between long-run changes in wealth-to-income ratios and relative productivity improvements in the rest of the economy, with respect to the construction sector, estimated using O'Mahony and Timmer (2009)'s KLEMS data.

While the results in terms of evolution of wealth and inequality have been already analyzed in the literature, to the best of our knowledge we are the first to uncover the link between long-run changes in wealth ratios and relative labor efficiency improvements.

## 2.1 Long-Run Trends

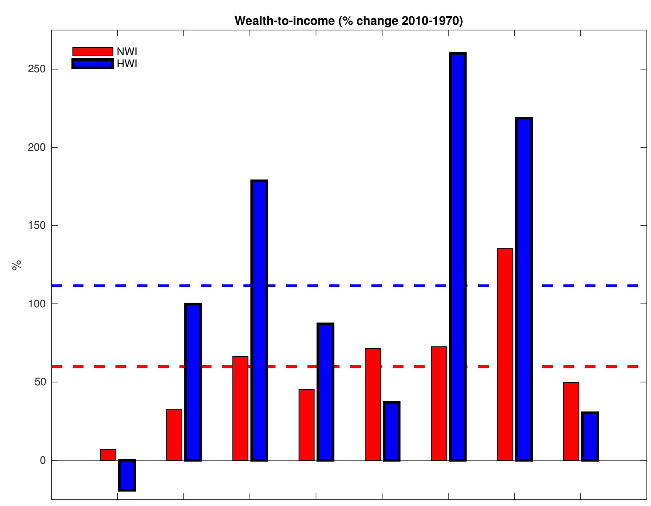
Piketty and Zucman (2014) have put together an incredibly rich dataset on wealth and income, starting from national accounts data, for the period 1970–2010, for the largest eight developed economies: the United States, Germany, the United Kingdom, Canada, Japan, France, Italy and Australia[4]. All assets and liabilities are valued at prevailing market prices. Private wealth is net wealth of households, and assets include all non-financial and financial assets. Public wealth is net wealth of public administrations and government agencies. National wealth is the sum of private and public wealth. While the financial component of private wealth includes households' holdings of domestic public debt, at the national level holdings of domestic debt are netted out. Housing wealth is one of the components of total wealth, and it measures the net value of households' real-estate holdings. As public debt should not be part of individuals' net (of the present value of future taxes) wealth over the long run, in this section we focus on national, rather than private, wealth, and on one of its main component, namely housing wealth, while we present data on private wealth in a separate online appendix.

In the eight largest economies, national wealth increased substantially more than income over the period 1970 to 2010. Figure 1 shows that, on average, the national wealth-to-income ratio increased by about 60 percent (red horizontal dashed-line). Interestingly, housing wealth increased even more, on average by about 112 percent (blue horizontal dashed-line). There exist important cross-country differences. For example, Italy is the country with the largest increase in the ratio between both national

[4]For a smaller subset of countries, Piketty and Zucman (2014) provide longer time-series. However, we choose to restrict our focus on a time-period for which we could maximize the number of countries in the sample and with more reliable data. For additional details on the data refer to the online appendix to this paper or directly to Piketty and Zucman (2014).

and housing wealth-to-income: 135 and 218 percent, respectively. On the contrary, in the US the national wealth-to-income ratio increased only by 6 percent, while housing wealth-to-income decreased by about 19 percent. In a separate appendix we show that these figures are robust to ending the sample in 2007, before the Great Recession, and to computing percentage changes using five-year averages at the beginning and end of the sample. The US are a bit of an exception. First, the adverse effect on national wealth of the Great Recession has been particularly severe. In fact, the wealth ratio increased by 36 percent if the sample ends in 2007. Second, the US, and Germany, are the countries with the largest differences between changes in national, as opposed to private, wealth-to-income. This depends on the fact that national wealth is pushed down by the larger shares of government debt held abroad. For example, in the US and Germany the private wealth ratio increased by 20 and 83 percent, respectively, in the period 1970–2010 (the average increase, across the eight largest economies, is 84 percent).

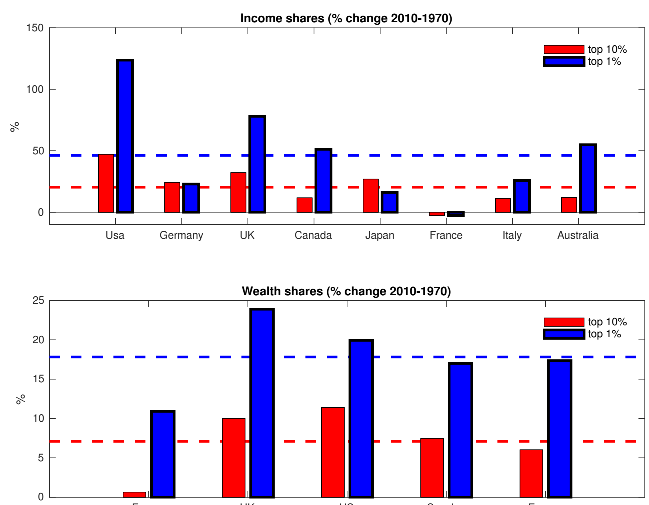
Figure 1: Wealth-to-Income Ratios (percentage changes 2010–1970)




*Notes:* This figure plots the percentage changes in national (NWI) and housing (HWI) wealth-to-income ratios over the sample 2010–1970. For Australia the sample is 2010–1978. Horizontal dashed-lines correspond to the cross-sectional averages (60 and 112 percent respectively). Both national and housing wealth are annual, at market prices, from Piketty and Zucman (2014). Additional details on the data are available in a separate online appendix.

Since wealth is typically unevenly distributed, it is not surprising that, over the same 1970 to 2010 period, income and wealth inequality increased along with the wealth-to-income ratio. Figure 2 shows that, on average, the shares going to the top 1 and 10 percent of the income distribution increased, respectively, by 46 and 20 percent. Similarly, also the shares going to the top 1 and 10 percent of the wealth distribution increased, by approximately 18 and 7 percent. While the US is the country with the smallest increase in the wealth-to-income ratio, it is also the country with the largest increase in both income and wealth inequality. In fact, the shares going to the top 1 and 10 percent of the income distribution increased, respectively, by 123 and 47 percent; those going to the top 1 and 10 percent of the wealth distribution increased, respectively, by 20 and 11 percent.

Figure 2: Income and Wealth Shares (percentage changes 2010–1970)




*Notes:* This figure plots the percentage changes, over the sample 2010–1970, of the income (top panel) and wealth (bottom panel) shares of the the top 10 (red bars) and 1 (blue bars) percent of the distribution. The income shares are for the US, Germany, the UK, Japan, France, Italy and Australia. The wealth shares are for France, the UK, the US, Sweden and Europe. Data on income (wealth) shares are annual (decennial) from Alvaredo et al. (2016)'s Top World Income Database. For the UK the sample is 2008-1971; for Sweden 2000-1970; for Italy 2005–1970. Additional details on the series are available in a separate online appendix.

## 2.2 A "Simplified" Housing Cost Disease

The reported large increase in wealth ratios and inequality is not a novel result. In fact, it is at the center of both the academic and the political debate. We are rather interested in understanding the potential explanations behind these facts and the conditions that make them more likely to occur. The main conjecture investigated in this paper is the positive effect on wealth ratios and inequality, with a particularly strong effect on housing wealth, of relative productivity improvements in the rest of the economy with respect to construction sector: this is what we refer to as the *housing cost disease*. To gain the main intuition, it is useful to start with a simplified model, with no capital and two sectors, along the lines of Baumol (1967). Denote with $a$ the exogenous labor productivity in manufacturing and assume that both the manufacturing and the housing sectors use labor as the only input, *i.e.*,

$$Y^m = aL^m, \qquad Y^h = L^h,$$

where $Y^j$ denotes the sector-specific output and $L^j$ the sector specific labor for $j = h, m$. By perfect competition, profit maximization, and perfect labor mobility, it is straightforward to derive:

$$W = q^h = a,$$

where $W$ is the market real wage, $q^h$ the (relative) price of a unit of housing and $a$ captures relative labor efficiency. Denoting with $L$ the total workforce, per-capita income is simply:

$$y = (Y^m + q^h Y^h)/L = (aL^m + aL^h)/L = a.$$

Assume that the economy is in a steady state with a constant per-capita demand of the housing stock, $h^d$, and denote with $n$ and $\delta$ the per-period population growth and housing depreciation rates. Then,

$$Y^h/L = (n+\delta)h^d.$$

A natural assumption is that $h^d = h^d(\pi, W)$, with $\pi$ denoting the user cost of housing, *i.e.*, the cost of a unit of housing net of the discounted (un-depreciated) resale value. Since we are at steady state, housing prices are constant and we have

$$\pi = q^h - (1-\delta)q^h/(1+r) = q^h\left(\frac{\delta + r}{1+r}\right)$$

for some exogenously fixed rate of interest, $r$. Since housing is the only source of households' wealth and $q^h = y = a$, the wealth-to-income ratio is:

$$\beta(a) \equiv \frac{q^h h^d}{y} = h^d\left(a\left(\frac{\delta+r}{1+r}\right), a\right),$$

Then, the effect of a rising relative productivity on the wealth-to-income ratio depends on the elasticities of housing demand with respect to its own price ($\hat{h}^d_\pi$) and wage income ($\hat{h}^d_W$)[5]. In particular, the percentage change in $\beta$ generated by a one percent increase in $a$ is equal to the sum of these two elasticities, *i.e.*,

$$\frac{\partial\beta/\beta}{\partial a/a} = \hat{h}^d_\pi + \hat{h}^d_W.$$

Observe that, with an homothetic representation of preferences, $\beta$ increases with $a$ if and only if housing demand is inelastic with respect to its own price, *i.e.*, $-\hat{h}^d_\pi < 1$. This condition squares with most empirical estimations. In fact, the general consensus in the existing literature on housing demand is that both income and price elasticities are relatively small in absolute value, the first ranging between 0.5 to 1, and the

[5]From now on, to simplify the notation, we use a "hat" to denote partial elasticities, *i.e.*, letting $h(x)$ be any differentiable function in $\mathbb{R}^n$, we let $\hat{h}_{x_i} = \partial \log h(x)/\partial \log x_i$.

second between -1 and -0.5 (for example, Mayo (1981) and Hansen et al. (1996)). In what follows, we show that this basic intuition holds in a more sophisticated life-cycle economy with capital and land. In particular, we consider a more general two-sector model with production functions

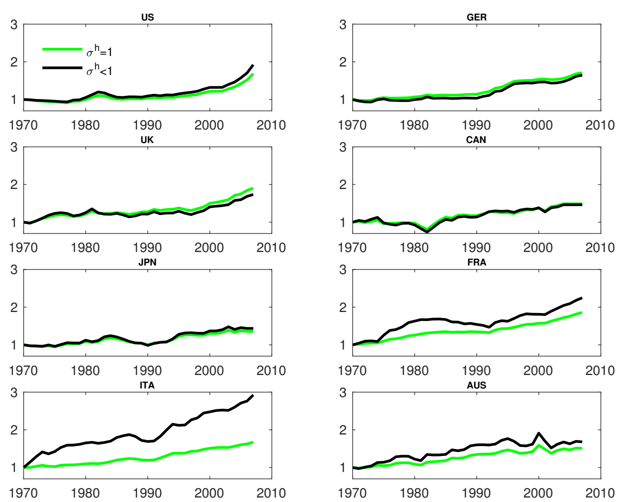
$$Y_t^h = F^h(K_t^h, A_t^h L_t^h, Z_t), \quad Y_t^m = F^m(K_t^m, A_t^m L_t^m), \tag{1}$$

where, for $j = h, m$, $K^j$ and $L^j$ are the amounts of capital and labor employed in the two sectors, $A^j$ is a labor-augmenting technological level and $Z$ represents the stock of land and government permits for residential development. For analytical convenience, we assume that both $F^h$ and $F^m$ exhibit constant returns to scale and we restrict $F^h$ to the class of CES productions functions. Embedding this technologies into an overlapping generations model with bequests, we will show that, under some general assumptions on preferences, the wealth-to-income ratio does not depend on the relative labor productivity of manufacturing when both the housing demand own price elasticity and the elasticity of substitution between factor inputs in construction are equal to one. On the other hand, a relatively inelastic housing demand, paired with an increase in relative labor productivity, generates a rise in the wealth-to-income ratio and in wealth inequality.

## 2.3 Some Preliminary Evidence

To provide some initial empirical support to our claim, we combine data on wealth ratios, for the eight largest advanced economies, with data on relative productivity in the rest of the economy with respect to construction, estimated from the O'Mahony and Timmer (2009)'s EU KLEMS Growth and Productivity Accounts. Even though in the model, for the sake of simplicity, we consider just two sectors, manufacturing and construction, for this section we construct a broader measure of relative productivity since manufacturing is only a small share of total value added (*i.e.*, approximately 13% in the US). In particular, we estimate relative productivity improvements by feeding data on gross value added, capital and labor inputs, to the production functions of each sector defined in (1) and considering "Total industries" (TOT) as the empirical counterpart for the rest of the economy[6]. In particular, relative labor efficiency is estimated under the assumption of Cobb-Douglas production function in the general economy, and CES with elasticity smaller than unity in construction. In figure 3 we plot, for the eight largest advanced economies and for the period 1970–2007, the estimates for the relative labor efficiency in the rest of the economy with respect to construction, $a$, for both the case of unitary elasticity of substitution in housing production (green line), and elasticity smaller than one (black line).

Figure 3: Relative Labor Efficiency in Manufacturing

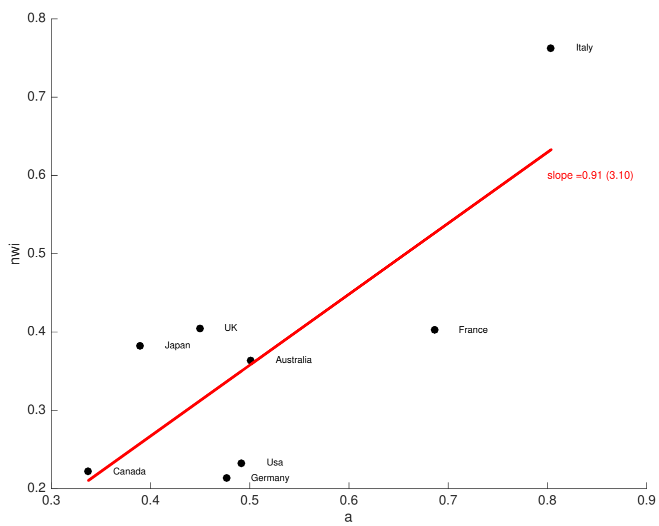



*Notes:* This figure plots two measures of relative labor efficiency in the rest of the economy, with respect to the construction sector, for the US, Germany, the UK, Canada, Japan, France, Italy and Australia for the period 1970–2007. Relative labor efficiency is estimated as residual assuming Cobb-Douglas technology in manufacturing, and either Cobb-Douglas (green lines) or CES (black lines) technology in housing construction. The elasticity of substitution for the CES technology is set to $\sigma^h = 0.6$. All series are normalized to 1 in 1970. Data are annual from O'Mahony and Timmer (2009). For each country we use the longest available series and extend it to the 1970–2007 sample setting any missing values to the closest available observation.

[6]In the KLEMS's data, "total industries" include all the sectors of the economy. Note that, following the technological assumptions spelled out in section 3, we assume that productivity is labor-augmenting, rather than affecting all the factors of production. Therefore, we are implicitly capturing relative changes in the quality of labor in different sectors of the economy.

In figure 4, we plot long-run log changes in national wealth-to-income against long-run log changes in relative productivity improvements. The sample starts in 1970 and ends in 2007, the last year of coverage of KLEMS. There exists a clear positive relationship between changes in relative labor efficiency and national wealth, with an estimated elasticity of 0.91 percent. In the separate online appendix to this paper, we show that the relationship between housing wealth, one of the main component of total wealth, and relative productivity improvements is even stronger, with an estimated elasticity of 1.8 percent.

Figure 4: Wealth ratios and relative labor efficiency




*Notes:* This figure plots long-run log changes in national wealth-to-income against long-run log changes in relative labor efficiency in the rest of the economy with respect to construction for the US, Germany, the UK, Japan, France, Italy and Australia. The log changes are computed between the average values of each variable for the periods 2007–2004 and 1974–1970. Relative labor efficiency is estimated from O'Mahony and Timmer (2009)'s KLEMS data assuming Cobb-Douglas technology for the manufacturing sector and CES technology with elasticity of substitution equal to $\sigma^h = 0.6$ for the construction sector. The red line corresponds to OLS fitted values. We also report estimates for the OLS slope (t-stats in brackets). Data for national wealth-to-income are from Piketty and Zucman (2014). Additional details on the series are available in a separate online appendix.

Our empirical findings do not prove any causal link, but rather the existence of co-movements, and are subject to several issues. First, the sample's small number of countries and limited time-length. Second, possible measurement errors in both Piketty and Zucman (2014) and O'Mahony and Timmer (2009)'s data and our measure of relative labor efficiency. Third, we do not control for alternative explanations of the long-run increase in wealth ratios which we consider complementary, rather than competitive, to our story. In the separate online appendix we address some of these issues, for example by considering the relationship between long-run changes in housing prices, a key driver of housing wealth, and relative productivity improvements, for a larger sample of sixteen OECD countries; and by performing robustness checks with respect to the computation of the long-run changes. In both cases, results confirm those presented in this section. Also note that, even though our estimates of relative labor productivities depend on the model's technology assumptions and overlook potential endogeneity issues, for our results to hold we mostly need to correctly capture the ranking of countries in terms of long-run changes in relative productivity rather then their exact magnitude.

# 3 The Model

In this section we present a simple life-cycle model with bequests, two sectors (construction and manufacturing), three assets (business capital, housing and land), and exogenous technical progress. The manufacturing sector stands to represent the non-construction sectors of the economy. Both sectors use labor and capital, and land is used in construction only. Assuming that capital and labor are perfectly mobile across sectors and firms are competitive, we identify conditions on technologies and households' preferences compatible with the stylized facts presented in section 2. We solve the model analytically and show that, when housing demand is sufficiently inelastic and the land share of output is sufficiently small, a rise in labor efficiency in manufacturing generates a set of phenomena that we label the *housing cost disease*. Namely, for a plausible parametrization of the model, there is a sizable increase in the steady state values of total and housing wealth-to-income ratios, bequests (*i.e.*, wealth inequality), housing and land prices, and share of labor in construction. It is worth noticing that housing wealth increases much more than total wealth and this is mainly due to a price effect. Furthermore, bequests rise substantially (and more than proportionally) and, contrary to the model with labor as the only input, average labor productivity increases less than proportionally (*i.e.*, the housing cost disease generates a stagnant measured average productivity). Finally, these phenomena are all compatible with a modest increase in the labor share employed in the construction sector.

## 3.1 Production

Technology is defined by the two production functions defined in (1) and satisfies the following assumption.

**Assumption 1.** *Both $F^h$ and $F^m$ display constant returns to scale, are strictly increasing, strictly concave, continuously differentiable and such that*

$$\lim_{K^j/A^jL^j \to 0} F_2^j/F_1^j = 0, \qquad \lim_{K^j/A^jL^j \to \infty} F_2^j/F_1^j = \infty,$$

*where $F_i^j$ denote the partial derivatives of $F^j$ with respect to the $i$-th argument. More specifically, $F^h$ belongs to the class of CES productions functions with elasticity of substitution between inputs defined by $\sigma^h$.*

We let the stock of land and government permits for residential development $Z$ be time-dependent because the government, or some other public authority, may decide to change it to enact specific policies or respond to demographic variables. More specifically, for some value $\xi > 0$, we let

$$Z_t = \xi L_t. \tag{2}$$

To simplify the exposition, we assume that the proceeds from selling land permits are used by the government to finance wasteful government spending. Note that we are implicitly assuming that past flows of new housing do not reduce the land stock available for the current production of housing. Therefore, our definition of housing production is not limited to new buildings on previously unused land, but includes rebuilding, renovation and construction of new floors of existing houses. It is straightforward to show that our assumption is almost isomorphic to the alternative assumption used, for example, by Davis and Heathcote (2005) and Favilukis et al. (2015), that considers a production function for new housing where the contribution of land depends on the flow of new land and government permits generated at any period[7].

It is convenient to provide a more compact notation by normalizing variables with respect to the level of labor efficiency. In particular, for $j = h, m$, let $k^j = K^j/A^jL^j$ be the sector-specific capital intensities, $z = Z/A^hL^h$ the available land per unit of labor efficiency in construction and $y^j$ the sector-specific labor productivities in efficiency units. By constant returns to scale

$$y^h = F^h(k^h, 1, z) \equiv f^h(k^h, z) \qquad y^m = F^m(k^m, 1) \equiv f^m(k^m),$$

where $f_j$ denotes the intensive-form production functions.

We further assume that firms in construction and manufacturing are price-takers and labor and capital are fully mobile across the two sectors. Let $a = A^m/A^h$ be the labor augmenting efficiency in manufacturing relative to construction (henceforth *relative productivity*); $r$ the real interest rate; $w = W/A^m$ the wage rate per units of efficiency in the manufacturing sector; and denote with $f_k^h$ and $f_z^h$ the partial derivatives of $f^h(k^k, z)$ with respect to $k^h$ and $z$, respectively. Then, letting $q^h$ be the price of a unit of new housing, profit maximization at any interior solution (*i.e.*, strictly positive

[7] In particular, if we interpret $Z_t$ as the flow of new land available for construction at time $t$, as in Davis and Heathcote (2005) and Favilukis et al. (2015), a strictly positive population growth rate, $n$, implies, asymptotically, a constant per-capita stock of land $\xi/n$. In our model, instead, the per capita stock of land is equal to $\xi$ at all $t \geq 0$.

$(k_t^j, L_t^j, y_t^j)$ for $j = h, m)$ implies

$$1 + r = f_k^m = q^h f_k^h, \tag{3}$$

$$w = f^m - k^m f_k^m = (q^h/a)[f^h - k^h f_k^h - z f_z^h]. \tag{4}$$

By the properties of the production functions, the above two equations provide a well defined map from $(r, a)$ into $(w, k^h, k^m)$ for all $a > 0$ and $r$ in a suitable interval. Importantly, the CES representation for $F^h$ implies that the profit maximizing values of the capital-labor ratios under factor price equalization are independent of the amount of land, $Z$. In particular, we can state the following.

**Proposition 1.** *For any given strictly positive $(r, a)$, with $r \in \mathcal{A} = [\underline{r}, \bar{r}]$, there is a unique solution, $(w(r), k^h(r, a), k^m(r))$, to equations (3)-(4), as a differentiable function of $(r, a)$, such that $w$, $k^h$ and $k^m$ are all decreasing in $r$ and*

$$\hat{k}_a^h \equiv \frac{\partial k^h / k^h}{\partial a / a} = \sigma^h. \tag{5}$$

A sketch of the proof is the following. Since $f_k^m(k^m)$ is decreasing in $k^m$, the marginal productivity of capital in manufacturing is locally invertible in some interval $\mathcal{A} = [\underline{r}, \bar{r}]$. Then, by the profit maximization condition (3) we obtain $k^m = k^m(r)$, with $k^m(.)$ decreasing in $r$ and such that $k^m(\underline{r}) = \infty$, $k^m(\bar{r}) = 0$. Existence of the function $w(r)$ follows from (3) and $k^m = k^m(r)$, which proves also $w'(r) < 0$. Now observe that, since the CES production function is homothetic, the ratio between the marginal product of labor and the marginal product of capital in the construction sector depends on $k^h$ only and

$$\frac{f^h(k^h, z) - k^h f_k^h(k^h, z) - z f_z^h(k^h, z)}{f_k^h(k^h, z)} = \omega_h(k^h),$$

where $\omega_h(k^h)$ is increasing and invertible in $\mathbb{R}_+$. By factor price equalization,

$$\omega_h(k^h) = aw(r)/r.$$

and, then, $k^h = \omega_h^{-1}(aw(r)/(1+r)) \equiv k^h(r,a)$. By taking the derivative, we obtain (5). Notice that the fact that $(w, k^h, k^m)$ are independent of $z$ is a consequence of the CES specification of $F^h$. By equations (3)-(4) and the above findings we can write the housing price as

$$q^h = \frac{(1+r)}{f_k^h\left(k^h(r,a), z\right)} \tag{6}$$

for all $(r, z, a)$ in $\mathcal{A} \times \mathbb{R}_+^2$.

## 3.2 Households

A set $L_t$ of households, growing at a rate $n \geq 0$, is born every period $t = 0, 1, 2, ...$ They live for two periods, supply labor time inelastically, in young age only, and have identical time-invariant preferences for manufacturing consumption and housing services, the latter being measured by the housing stock. Households are characterized by some degree of altruism with respect to their offsprings defined by an individual specific discount rate of the next generation's utility. In particular, households born at time $t$ belong to different types, indexed by $i$, with $i$ in a finite set $\mathcal{I}$, and each type $i$ composed of a mass $m_i$ of individuals (*i.e.*, a collection of positive numbers, $(m_i)_{i \in \mathcal{I}}$, such that $\sum_{i \in \mathcal{I}} m_i = 1$), with life-time utility defined by:

$$V^{t,i} = u(c_t^{t,i}, c_{t+1}^{t,i}, h_{t+1}^i) + \theta_i(1+n)V^{t+1,i},$$

for all $t \geq 0$, where $(c_t^{i,t}, c_{t+1}^{i,t})$ are age-contingent consumptions, $h_{t+1}^i$ is the housing stock acquired by the household in young age. Preferences satisfy the following assumption.

**Assumption 2.** *The (inter-generational) discount factors satisfy* $\theta_i(1+n) < 1$ *for all*

*$i \in \mathcal{I}$ and the utility function belongs to the CES class with elasticity of substitution between goods denoted by $\gamma$.*

In particular, we specify utility as:

$$u(c^{y,i}, c^{o,i}, h^i) = \begin{cases} \left[\chi^y (c^{y,i})^{\frac{\gamma-1}{\gamma}} + \chi^o (c^{o,i})^{\frac{\gamma-1}{\gamma}} + \chi^h (h^i)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}} & \text{if } \gamma \neq 1, \\ \chi^y \log c^{y,i} + \chi^o \log c^{i,o} + \chi^h \log h^i & \text{otherwise,} \end{cases} \tag{7}$$

where $\chi^j > 0$ for $j = y, o, h$ and $\sum_{j=y,o,h} \chi^j = 1$.

The upper bound on the discount rates, $\theta_i$, insures convergence of each dynasty's long-run utility function. Importantly, the CES hypothesis guarantees normality, unitary elasticity of consumption and housing demand with respect to income, and the "law of demand" (in particular, the demand for housing is decreasing in its own price).

We assume perfect financial markets allowing for unlimited lending and borrowing and, for simplicity, we ignore the housing rental market[8]. Any household born at time $t$ acquires land and residential property when young, enjoys the housing services generated by it, resells the property when old and leaves some bequests to the offsprings. Due to the absence of financial frictions, we can write the households' inter-temporal budget constraints as:

$$c_t^{t,i} + \frac{c_{t+1}^{t,i}}{1 + r_{t+1}} + \pi_t h_{t+1}^i + \frac{(1+n) b_{t+1}^i}{1 + r_{t+1}} = W_t + b_t^i, \tag{8}$$

where $b_t^i$ denotes bequests, $W_t$ is the time-$t$ real wage and

$$\pi_t = q_t^h - (1-\delta) q_{t+1}^h / (1 + r_{t+1})$$

[8]Households are assumed to derive some, however small, satisfaction from ownership, so that, in the absence of market frictions, ownership is a dominant choice relative to renting. The average home ownership rate across OECD countries is approximately 67%. At the top of the distribution are countries like Greece (87%) and Spain with high rates of more than 80%; while at the bottom countries like Germany (43%), Japan (36%) and Switzerland (35%). Data are from the OECD (2012).

denotes the user cost of housing, *i.e.*, the cost of a unit of housing net of the present value of selling the same un-depreciated unit the next period and $\delta \in (0,1)$ is the housing depreciation rate. Notice that the above representation of the inter-temporal budget constraint takes into account the absence of arbitrage opportunities related to the assets included in households' portfolios, such as bonds, housing and land. Since parental altruism is one-sided, we rule out forced gifts from children to parents, and impose the non-negativity constraint

$$b^i_{t+1} \geq 0.$$

Denoting with $u^i_{j,t}$ the partial derivative of $u(c^{t,i}_t, c^{t,i}_{t+1}, h^i_{t+1})$ with respect to the $j$-th argument and with $q^z$ the price of a unit of land, the first order conditions characterizing the young household's optimal consumption choice subject to the budget constraint, (8), are

$$u^i_{1,t} = (1 + r_{t+1})u^i_{2,t}, \qquad u^i_{1,t}\pi_t = u^i_{3,t}, \qquad u^i_{2,t} \geq \theta_i u^i_{1,t+1}, \tag{9}$$

together with the complementary slackness condition

$$b^i_{t+1}\left(u^i_{2,t} - \theta_i u^i_{1,t+1}\right) = 0 \tag{10}$$

and the no-arbitrage condition related to land investment,

$$q^z_t = (q^h_{t+1} f^h_z(k^h_{t+1}, z_{t+1}) + q^z_{t+1})/(1 + r_{t+1}), \tag{11}$$

where $q^z$ denotes the unit price of land. From the above first order conditions we derive the time-$t$ households saving, $s^i_t$, the demand for housing, $h^i_{t+1}$, and the supply of bequests, $b^i_{t+1}$. These are specified as:

$$
\begin{aligned}
s^i_t &= S^i(\pi_t, r_{t+1}, W_t + b^i_t), & (12)\\
h^i_{t+1} &= H^{d,i}(\pi_t, r_{t+1}, W_t + b^i_t), & (13)\\
b^i_{t+1} &= B^i(\pi_t, r_{t+1}, W_t + b^i_t). & (14)
\end{aligned}
$$

## 3.3 Equilibrium

Letting $K$ and $L$ be the total stocks of business capital and labor, full employment implies

$$
\begin{aligned}
L_t &= L^h_t + L^m_t, & (15)\\
K_t &= K^h_t + K^m_t, & (16)
\end{aligned}
$$

and housing, $H$, evolve over time according to the following conditions

$$H_{t+1} = Y^h_t + (1-\delta)H_t. \tag{17}$$

With no loss of generality we impose the normalization $A^m = 1$ and express all equilibrium restrictions and the relevant variables in per-capita units. In particular, letting

$$k_t = K_t/L_t, \quad h_t = H_t/L_t, \quad \lambda_t = L^h_t/L_t,$$

we replace the full employment conditions (2), (15), (16) and (17) with

$$
\begin{aligned}
k_t &= \lambda_t k^h_t + (1-\lambda_t)a_t k^m_t, & (18)\\
z_t &= \xi/\lambda_t, & (19)\\
(1+n)h_{t+1} &= \lambda_t y^h_t + (1-\delta)h_t & (20)
\end{aligned}
$$

and $\lambda_t \in [0,1]$. To close the model we define the per capita aggregate saving and housing demand

$$\begin{aligned} s_t &\equiv \sum_i m_i S^i(\pi_t, r_{t+1}, W_t + b_t^i), \\ h_{t+1}^d &\equiv \sum_i m_i H^{d,i}(\pi_t, r_{t+1}, W_t + b_t^i)/(1+n), \end{aligned}$$

and impose market clearing in the the housing and capital markets, *i.e.*,

$$h_{t+1} = h_{t+1}^d, \tag{21}$$

$$s_t = (1+n)(k_{t+1} + q_t^h h_{t+1} + q_t^z \xi). \tag{22}$$

By exploiting proposition 1, *i.e.*, profit maximization and factor price equalization,

$$k_t^m = k^m(r_t), \quad k_t^h = k^h(r_t, a_t), \quad W_t = a_t w(r_t), \quad q_t^h = (1+r_t)/f_k^h(k_t^h, z_t). \tag{23}$$

Remember that the functions in (23) are positive and continuous and guarantee an interior allocation of factors across sectors for $r$ in $\mathcal{A} = [\underline{r}, \bar{r})$ and all $a > 0$. Then, an *interior competitive equilibrium* is a positive sequence,

$$\{k_t, k_t^h, k_t^m, \lambda_t, h_t, b_{t+1}, r_{t+1}, w_t, q_t^h, q_t^z\}_{t=0}^{\infty},$$

with $r_{t+1} \in \mathcal{A}$ and $\pi_t = q_t^h - (1-\delta)q_{t+1}^h/(1+r_{t+1}) > 0$ for all $t \geq 0$, satisfying the optimality conditions (9)-(11), the market clearing conditions (18), (19), (20), (21), (22), the factor price equalization conditions (23), for all $t \geq 0$, for a given sequence of relative productivities, $\{a_t\}_{t=0}^{\infty}$, and some initial conditions, $(k_0, h_0, b_0, r_0) > 0$. We neglect the government balanced budget condition since it has no impact on equilibrium variables[9].

[9]Remember that we are assuming that the proceeds from selling land permits are used for wasteful government spending.

## 3.4 Steady States

From now on we concentrate on a steady state equilibrium with two types of households. In particular, letting $\mathcal{I} = \{p, r\}$ and $\theta_r > \theta_p$, we say that household type $r$ is *rich* and household type $p$ is *poor*, although we could as well say that the former is more altruistic than the latter with respect to their own children. Under this specification, and by the first order conditions (9)–(10), it is clear that we may have two type of steady states. In the first, $r \leq (1-\theta_r)/\theta_r$ and no individual leaves any bequests, so that the resulting equilibrium is equivalent to the one that would take place in a *canonical* overlapping generations economy. In the second, the rich individuals leave positive bequests, whereas the poor leave zero bequests at any time. In this case we have

$$r = (1-\theta_r)/\theta_r \equiv r^* < (1-\theta_p)/\theta_p.$$

We refer to the first type of equilibrium as a *zero bequests steady state* (ZBSS) and to the second type as a *positive bequests steady state* (PBSS). In what follows, we focus exclusively on the PBSS, mostly because this type of equilibrium allows for a sharp characterization of intra-generational inequality. It is understood that a PBSS is assumed to be *interior*, *i.e.*, to imply that both sectors are active. In particular, we impose the following assumption.

**Assumption 3.** $\lim_{k^m \to 0} f_k^m > \min_{i \in \mathcal{I}} 1/\theta_i > \lim_{k^m \to \infty} f_k^m$.

The above implies that a PBSS exists and it is such that the wage rate per unit of efficiency is uniquely fixed at $w^* = w(r^*)$, whereas $w \geq w^*$ at a steady state with zero bequests. Recalling that any interior allocation provides $k^m = k^m(r)$, $k^h = k^h(r, a)$, the allocation of the capital-labor ratios is uniquely determined by $(r^*, a)$. Hence, there are six remaining equilibrium variables to be determined by the steady state equilibrium conditions characterizing a PBSS: the average capital stock, $k$, the share of labor in construction, $\lambda$, the housing stock, $h$, the asset prices, $q^h$, $q^z$, and the steady state bequest of the rich household, $b^r$, all of them uniquely determined by $(r^*, a)$. Since we are mostly interested in the impact of the relative productivity parameter $a$, we define a PBSS as an array

$$\mathcal{E}^*(a) = (k^*(a), k^{m,*}, k^{h,*}(a), \lambda^*(a), h^*(a), b^{r,*}(a), w^*, q^{h,*}(a), q^{z,*}(a)).$$

We now review some key features and properties of our model economy at a PBSS that are relevant for understanding how and when a cost disease may occur. Dropping the superscript "$*$" on the PBSS equilibrium values of the interest rate, wage and capital, equation (20) provides the following relation

$$(\delta + n)h = f^h(k^h(r, a))\lambda, \tag{24}$$

and, by profit maximization, the no-arbitrage condition (11) and the land policy (19), steady state asset prices are

$$\begin{aligned} q^h &\equiv (1 + r)/f_k^h(k^h(r, a), \xi/\lambda), &(25)\\ q^z &\equiv q^h f_z^h(k^h(r, a), \xi/\lambda)/r, &(26)\\ \pi &= q^h(\delta + r)/(1 + r). &(27) \end{aligned}$$

Regarding households' demand of consumption and housing, we observe that, at a PBSS, these are the solutions to the maximization of the "instantaneous utility function", $u(c^{y,i}, c^{o,i}, h^i)$, subject to the (present value) budget constraint

$$c^{y,i} + \frac{c^{o,i}}{1 + r} + \pi h^i = aw + \left(\frac{r - n}{1 + r}\right) b^i \equiv I^i, \tag{28}$$

and we denote them as $c^{y,i}(\pi, aw, I^i)$, $c^{o,i}(\pi, aw, I^i)$, $h^i(\pi, aw, I^i)$. Hence, since households' saving at steady state is

$$s^i(\pi, aw, I^i) \equiv aw + b^i - c^{y,i}(\pi, aw, I^i),$$

we define the aggregate demand for housing and aggregate saving as

$$h^d(\pi, aw, b) \equiv \sum_i m_i h^i(\pi, aw, b^i)/(1+n), \tag{29}$$

$$s(\pi, aw, b) \equiv \sum_i m_i s^i(\pi, aw, b^i), \tag{30}$$

where, to simplify the notation, we have set $b = b^r$ (as $b^p = 0$ at the PBSS, there is no confusion regarding the identity of bequests). By the CES utility specification, demands for consumption and housing are homothetic, satisfy the law of demand (*i.e.*, housing and age-contingent consumptions are decreasing in their own price) and exhibit a unitary income elasticity. In particular,

$$c^{y,i} = \phi^y(\pi) I^i, \qquad \frac{1}{1+r} c^{o,i} = \phi^o(\pi) I^i, \qquad \pi h^i = \phi^h(\pi) I^i,$$

where $\phi^j(\pi) \in (0,1)$ (for $j = y, o, h$) are the expenditure shares defined by

$$\phi^y = \frac{(\chi^y)^\gamma}{(\chi^y)^\gamma + (\chi^o)^\gamma (1+r)^{\gamma-1} + (\chi^h)^\gamma \pi^{1-\gamma}}, \tag{31}$$

$$\phi^o = \frac{(\chi^o)^\gamma (1+r)^{\gamma-1}}{(\chi^y)^\gamma + (\chi^o)^\gamma (1+r)^{\gamma-1} + (\chi^h)^\gamma \pi^{1-\gamma}}, \tag{32}$$

$$\phi^h = \frac{(\chi^h)^\gamma \pi^{1-\gamma}}{(\chi^y)^\gamma + (\chi^o)^\gamma (1+r)^{\gamma-1} + (\chi^h)^\gamma \pi^{1-\gamma}}. \tag{33}$$

A further important property is that the elasticities of consumption and housing demand with respect to the user cost of housing are independent of individuals' wealth and given by

$$\hat{h}^i_\pi = \hat{c}^{y,i}_\pi = \hat{c}^{y,i}_\pi/(1+r) = -(1-\gamma)\phi^h,$$

where $\gamma$ is the elasticity of substitution between goods. Finally, since consumption and housing are normal goods, we conclude that the rich households' housing demand, $h^r$, and saving,

$$s^r \equiv aw + b^r - c^{y,r}(\pi, aw, I^r) = (1-\phi^y)aw + \left(\frac{1+(1-\phi^y)r+\phi^y n}{1+r}\right) b^r,$$

are both increasing in bequests. Hence, higher bequests generate more housing demand and more saving for the $r$-type individual relative to the $p$-type at the PBSS. An important related property that we will exploit to derive the comparative statics of the model, is that the impact of a rising bequest on saving is greater than the impact on the money spent on housing, *i.e.*,

$$\partial s^i/\partial b^i > q^h \partial h^i/\partial b^i > 0. \tag{34}$$

Notice that these inequalities are not specific of the CES utility representations, but rather hold also in a more general setting under the conditions of normality and $r > n \geq 0$.

These properties allow for an easy aggregation of individuals' demands and savings and their elasticities. Observe that the effect of a rising user cost of housing on the aggregate demand for housing is

$$\hat{h}^d_\pi = \frac{1}{1+n}\sum_i \left(\frac{m_i h^i}{h^d}\right)\hat{h}^i_\pi, \qquad \hat{s}_\pi = \sum_i \left(\frac{m_i s^i}{s}\right)\hat{s}^i_\pi,$$

and, then, by the CES specification,

$$1+\hat{h}^d_\pi = (1-\gamma)(1-\phi^h), \qquad \hat{s}_\pi = (1-\gamma)\phi^h c^y/s, \tag{35}$$

where $c^y = \sum_i m_i c^{y,i}$ is the aggregate young age consumption. Hence, if $\gamma < 1$, a rise in the user cost of housing generates, *ceteris paribus*, a rise in the demand for housing wealth and saving. On the contrary, if $\gamma = 1$, then $\hat{h}^d_\pi = -1$ and $\hat{s}_\pi = 0$.

# 4 The Housing Cost Disease

Using the results obtained in the last section, we can now explore under what conditions our model replicates some of the features of the two-sector economy studied by Baumol (1967), with construction of housing playing the role of the *stagnant sector* and manufacturing the role of the *dynamic sector*. We recall that the *Baumol's cost disease* holds if, following a rise in productivity in the dynamic sector, (a) the relative price of the stagnant sector output increases (*price increase*); (b) the stagnant industry takes a rising share of nominal output (*unbalanced growth*); and (c) the changing composition of output across stagnant and dynamic industries reduces the effect of the productivity improvement on the average productivity (*adverse effect on productivity*). We restate the Baumol's cost disease result in the present framework as follows.

Define *housing wealth* as $v \equiv q^h h$ and the average income per-capita as

$$y = ay^m(1-\lambda) + q^h y^h \lambda = aw + (1+r)k + rq^z\xi. \tag{36}$$

Then, the *wealth-to-income-ratio* is

$$\beta = \frac{k + v + q^z\xi}{y} = \beta^k + \beta^h + \beta^z,$$

where $\beta^k = k/y$ is the business capital, $\beta^h = v/y$ the housing, and $\beta^z = q^z\xi/y$ the land, component. Then, we say that there exists a *housing cost disease* if, at an equilibrium

PBSS,

$$\begin{array}{lll} (\textsc{ha}) & \partial q^h/\partial a > 0 & \text{(housing appreciation),} \\ (\textsc{iw}) & \partial \beta^h/\partial a > 0, \quad \partial \beta/\partial a > 0 & \text{(increasing wealth-to-income ratios),} \\ (\textsc{in}) & \partial b/\partial a > 0 & \text{(increasing wealth inequality),} \\ (\textsc{sp}) & \partial y/\partial a < y/a & \text{(stagnant labor productivity).} \end{array}$$

Observe that (IW) may be defined, alternatively, as an increase in the share of labor employed in construction. However, the two phenomena are strictly related by (24).

It turns out that the emergence of a cost disease in the present model depends crucially on the magnitude of relative capital intensities and factor shares. In the remaining part of this section, we first provide intuition and, then, some analytical results. In particular, define the sector-specific *factor shares* (or output elasticities):

$$\mathcal{S}_k^j = f_k^j k^j / f_j, \qquad \mathcal{S}_z^h = f_z^h z / f^h, \qquad \mathcal{S}_l^h = 1 - \mathcal{S}_k^h - \mathcal{S}_z^h.$$

Then, we let the *capital intensity differential* across the two sectors be defined as

$$\Delta = \left( \frac{\delta + n}{1 + r} \right) \mathcal{S}_k^h \left( \frac{a k^m - k^h}{k^h} \right).$$

Observe that $\Delta$ is a function of factor shares. In particular,

$$\frac{a k^m - k^h}{k^h} = \frac{1}{1 - \mathcal{S}_k^h} \left( \frac{\mathcal{S}_l^h (\mathcal{S}_k^m - \mathcal{S}_k^h)}{\mathcal{S}_k^h (1 - \mathcal{S}_k^m)} - \frac{\mathcal{S}_z^h (1 - \mathcal{S}_k^m)}{\mathcal{S}_k^m} \right). \tag{37}$$

Then, $\mathcal{S}_k^m > \mathcal{S}_k^h$ is a necessary condition for the manufacturing sector to exhibit a higher capital intensity[10], *i.e.*, $\Delta > 0$. In general, factor shares in the construction sector and $\Delta$ are functions of $\lambda$ and $a$. However, in the special case of Cobb-Douglas production in housing (*i.e.*, $\sigma^h = 1$), $\Delta$ is constant since both $\mathcal{S}_k^h$ and $\mathcal{S}_z^h$ are constant.

[10]Note that $\mathcal{S}_k^m > \mathcal{S}_k^h$ if and only if the housing price, $q^h$, is decreasing in the interest rate, $r$.

To appreciate the role of factor shares and capital intensities, observe that, by profit maximization and the supply of housing, (24), we derive

$$\lambda = ((\delta + n)\mathcal{S}_k^h/((1 + r)k^h))v, \tag{38}$$

$$q^z\xi = ((\delta + n)\mathcal{S}_z^h/r)v. \tag{39}$$

Then, using the above with (18) and (36),

$$k = ak^m - \Delta v, \tag{40}$$

$$y = ay^m - ((1 + r)\Delta - (\delta + n)\mathcal{S}_z^h)v, \tag{41}$$

$$\beta = (ak^m + ((\delta + n)\mathcal{S}_z^h/r + 1 - \Delta)v)/y. \tag{42}$$

Equations (40)–(42) provide some intuition about the role of the assumed restrictions on the relative capital intensity for the housing cost disease. In particular, when $\Delta$ is positive and $\mathcal{S}_z^h$ is sufficiently small, all else unchanged, a larger housing wealth, $v$, decreases $k$ and $y$, and increases $\beta^h$, both directly and through a fall in $y$. Provided that $\Delta$ is also not too large, a higher $v$ may, in turn, generate a larger wealth-to-output ratio.

In the remaining part of the present section we provide some analytical results imposing the following assumption.

**Assumption 4.** *The parameter values* $(\sigma^h, \mathcal{S}_l^h, \mathcal{S}_k^h, \mathcal{S}_z^h, \Delta, \delta, r, n)$ *satisfy:*

$$\sigma^h = 1, \tag{43}$$

$$1 + (\delta + n)\mathcal{S}_z^h/r > \Delta > (\delta + n)\mathcal{S}_z^h/r, \tag{44}$$

$$\mathcal{S}_l^h > \mathcal{S}_z^h. \tag{45}$$

The above restrictions are motivated by analytical tractability and by the objective of making the housing cost disease a likely outcome under some additional intuitive conditions. In particular, the restriction $\sigma^h = 1$ is mostly a simplifying assumption, as it implies that factor shares, $\mathcal{S}_j^h$ (for $j = l, k, z$), and the capital intensity differential, $\Delta$, are parametric. The upper bound on $\Delta$ in (44) is a requirement for a meaningful steady state equilibrium when bequests are not too large. In section A.III of the appendix we show, in fact, that this upper bound is necessary for generating $\lambda \leq 1$ at equilibrium, *i.e.*, non-negative shares of labor in the two sectors, under the assumption that aggregate saving falls short of the wage bill. This is a natural restriction, which is clearly satisfied when total bequests are not too large (and, *a fortiori*, in the canonical overlapping generations model). The lower bound on $\Delta$ (*i.e.*, a sufficiently large capital intensity in manufacturing relative to construction and a sufficiently low land share of income) is, instead, a key restriction for generating the rising wealth-to-income ratios that are part of the housing cost disease defined in this paper. A further motivation for concentrating on economies satisfying assumption 4 is that these properties imply a "well behaved" comparative statics for the system of equations defining a PBSS when the own price elasticity of housing demand is sufficiently close to one (*i.e.*, $\gamma \sim 1$). More intuition and details are in appendix A.III.

In the following proposition we provide some results about the elasticities of the relevant equilibrium variables with respect to $a$ under assumption 4.

**Proposition 2.** *Consider a PBSS, $\mathcal{E}^*(a)$, of an economy satisfying assumption 4. If $\gamma = 1$,*

$$\hat{b}_a^* = \hat{v}_a^* = \hat{y}_a^* = 1, \quad \hat{\beta}_a^* = \hat{\beta}_a^{h,*} = \hat{\lambda}_a^* = 0, \quad \hat{q}_a^{h,*} = 1 - \mathcal{S}_k^h, \quad \hat{q}_a^{z,*} = 1.$$

*If, on the other hand, $\gamma < 1$ and sufficiently close to 1, we have*

$$\hat{b}_a^* > 1, \quad \hat{v}_a^* > 1, \quad \hat{y}_a^* < 1, \quad \hat{\beta}_a^* > 0, \quad \hat{\beta}_a^{h,*} > 0, \quad \hat{\lambda}_a^* > 0, \quad \hat{q}_a^{h,*} > 1 - \mathcal{S}_k^h, \quad \hat{q}_a^{z,*} > 1.$$

Hence, the *unit-elastic* economy, defined by $\sigma^h = \gamma = 1$, is a benchmark case in which a change in $a$ leaves the shares of labor in the two sectors and the wealth-to-income ratios unchanged, increases average productivity and bequests on a one-to-one basis, and has a moderate effect on housing and land prices. On the other hand, when the housing demand is inelastic, all effects are magnified. In section A.III of the appendix we show that, when $\gamma$ is smaller than, but sufficiently close to, 1, house and land elasticities at equilibrium have the following lower bounds

$$\hat{q}_a^{h,*} > \frac{1 - \mathcal{S}_k^h}{1 - (1-\gamma)(1-\phi^h)\mathcal{S}_z^h}, \qquad \hat{q}_a^{z,*} > \frac{1 + (1-\gamma)(1-\phi^h)\mathcal{S}_l^h}{1 - (1-\gamma)(1-\phi^h)\mathcal{S}_z^h}. \tag{46}$$

Note that these lower bounds are increasing in the land share of income, $\mathcal{S}_z^h$, and decreasing in the (absolute value of the) own price elasticity of housing demand, $-\hat{h}_\pi^d$. Then, the more important is the fixed factor in the construction sector, the larger is the effect of relative productivity improvements on asset prices; and the more elastic is the housing demand, the smaller are the price effects. In addition, the lower bound on the equilibrium land price elasticity is higher than the one on the housing price, suggesting that land may have a strong role in the dynamics of wealth to income ratios. This result holds despite the assumption that the amount of land available for construction adjusts with population growth, *i.e.*, despite the absence of a sort of demographic pressure on land space.

What is the empirical support for the assumptions that we impose to generate a housing cost disease? In the literature, there is consistent support for the assumptions that the construction sector is less capital intensive than manufacturing and little consensus on the most plausible values for the elasticity of substitution between capital and labor. In section 4.1, we calibrate the model to match a series of stylized facts for the US and show that it can go a long way towards explaining a sizable fraction of the observed increases in wealth ratios and inequality. In any case, it is important to stress that the above assumptions are not necessary, bur rather they provide with the most favorable environment for the housing cost disease. In particular, the low price elasticity in the demand for the output of the stagnant sector is one of the key assumption in Baumol (1967)'s model. We observe that there exists strong evidence that housing demand responds less than proportionally to a rise in price. In particular, Hanushek and Quigley (1980), Mayo (1981) and Ermisch et al. (1996) provide estimates of the housing demand elasticity in the range $(-0.8, -0.5)$. Similarly, Ngai and Pissarides (2007) assumes a low (*i.e.*, below one) elasticity of substitution across final goods in order to show that employment is gradually shifting to sectors with low productivity growth. In our model, a rise in relative efficiency in manufacturing determines an increase in the relative price of housing with a small effect on the demand of this good. Therefore, the demand for housing wealth ($v = qh$) increases with $a$ and we observe a reallocation of production and labor to the less productive sector. The assumption that $\Delta > 0$ has no analogous counterpart in the literature following the Baumol's cost disease proposition.

We end this section with a remark on the behavior of factor shares under the cost disease phenomenon. Piketty and Zucman (2014) argue that the rising capital share of income is a consequence of the joint hypothesis of a steadily rising capital-output ratio and low diminishing returns to capital. Most of the evidence on the rising capital-output ratios and capital shares provided in this literature reflects the role of housing. For example, Rognlie (2014) claims that housing "accounts for nearly 100% of the long-term increase in the capital/income ratio, and more than 100% of the long-term increase in the net capital share of income." According to Bonnet et al. (2014), the capital income ratio has dropped or remained roughly constant, when we take housing capital aside. Therefore, it is interesting to evaluate whether the housing cost disease may be responsible for a higher capital share of income. We find that this is the case, but only when we consider a definition of capital share that includes imputed rents. In our model, the most natural definition of the capital share of income is

$$\zeta = 1 - aw/y.$$

Then, quite trivially,

$$\hat{\zeta}_a^* = \frac{aw}{y - aw}(\hat{y}_a - 1),$$

*i.e.*, the average capital share of income always falls in the presence of a housing cost disease, as wages grow proportionally with $a$ and average labor productivity grows by less than proportionally with $a$. However, if, following the prevailing national accounting practice, we include imputed rents in the definition of GDP the definition of capital share becomes:

$$\zeta^h = 1 - \frac{aw}{y + \pi h} = 1 - \frac{1 - \zeta}{1 + \left(\frac{\delta + r}{1 + r}\right)\beta^h}.$$

Hence, $\zeta^h$ is typically larger than $\zeta^k$ and it is increasing in $\beta^h$ for any given $y$. By straightforward computations we derive that, when $\hat{v}_a^* > 1$,

$$\hat{\zeta}_a^{h,*} \geq 0 \qquad \Leftrightarrow \qquad \Delta(1 + r) \leq \left(\frac{\delta + r}{1 + r}\right) + (\delta + n)\mathcal{S}_z^h.$$

By (44), the above condition requires $\Delta < (\delta + r)/(1 + r)$. Hence, when the definition of income includes imputed rents, a rising relative productivity may generate a rising capital share if the growth in housing wealth is sufficiently strong.

## 4.1 Numerical Simulation

In the previous sections, we showed analytically the effects of productivity improvements under the special assumptions of unit-elastic housing construction technology ($\sigma^h = 1$), and considering both the case of unit-elastic preferences, $\gamma = 1$, and of $\gamma < 1$ (cf. proposition 2). In this section, we evaluate the consequences of a rising relative efficiency in manufacturing on stationary equilibrium variables and show the impact of the housing cost disease on housing prices, wealth ratios, average productivity, and inequality under more general assumptions. In particular, we assume Cobb-Douglas technology in manufacturing, and CES with elasticity of substitution $\sigma^h$, possibly smaller than 1, in construction. In what follows, we express all variables in intensive form following the notation presented in the paper. The production functions for the two sectors are:

$$\begin{aligned} f^m &= (k^m)^{\alpha^m}, \\ f^h &= [\alpha^h (k^h)^{\frac{\sigma^h - 1}{\sigma^h}} + \eta^h z^{\frac{\sigma^h - 1}{\sigma^h}} + (1 - \alpha^h - \eta^h)]^{\frac{\sigma^h}{\sigma^h - 1}}. \end{aligned}$$

Table 1 reports all the parameters used in the numerical simulations. Preferences are of the CES class (cf. equation (7)). We set the coefficients of the CES utility function in order to match some stylized facts for the US economy. First, we set $\chi^h = 0.15$ to match the US households expenditure on housing services (approximately 15% of 2015 GDP according to the BEA NIPA Table 2.3.5). In order to set the coefficients attached to the consumption expenditure of young ($\chi^y$) and old ($\chi^o$) individuals we use the fact that, in the US in 2014, older people (aged 65 or more) are approximately 13% of the population (World Development Indicators from World Bank Data). Assuming that the consumption shares of young and old follow their shares in the population we set the weights $\chi^y$ and $\chi^o$, respectively, to 0.72 and 0.13. Note that the model implied expenditure shares are closely related to the coefficients $\chi$ of the CES preferences, but more precisely defined by equations (31)–(33). Following Ogaki and Reinhart (1998), we set the elasticity of inter-temporal substitution for the households' preferences, $\gamma$, to a number smaller than 1 and, in particular, equal to 0.5. Recall that in the PBSS, for the two-type of households' case (*i.e.*, rich and poor), described in section 3.3, the real interest rate is pinned down by the preference for altruism of the rich household. We set the real interest rate to match the average real return on US Treasury of about 1%

per year. Therefore, for a holding period of 25 years, corresponding to a generation, we set $r = 0.28$. The depreciation of the housing stock is set equal to $\delta = 20\%$, implying complete depletion over five generations as in Deaton and Laroque (2001). We use O'Mahony and Timmer (2009)'s KLEMS data to have rough estimates of the capital factor shares in construction and manufacturing in the US over the 1970–2007 period and, accordingly, set $\alpha^m = 0.35$ and $\alpha^h = 0.1$. These numbers are in line with those in Valentinyi and Herrendorf (2008) who set the capital share in manufacturing and construction respectively to 0.4 and 0.2. Therefore, the empirical evidence supports our assumption that the manufacturing sector is more capital intensive than the housing sector. Note that while technology in manufacturing is Cobb-Douglas, we assume a more general CES specification for the housing sector. Therefore, while $\alpha^m$ corresponds to the model implied capital share of manufacturing output, $\alpha^h = 0.1$ is only approximately corresponding to the capital share in housing production. Similarly, we set the weight attached to the land input to $\eta^h = 0.50$ following evidence in Knoll et al. (2014) that the land share of value added in construction is large, and close to 50%. Note that previous empirical work used lower values for the land share of output in construction: for example, Neels (1982) provide an estimate of the output elasticity of land (*i.e.*, our measure of $\mathcal{S}^h_z$) between 0.03 and 0.06, and Davis and Heathcote (2005) set this value at 0.106 for their own calibration. There is no consensus in the empirical literature on the value of the elasticity of substitution between capital and labor. For example, while Piketty and Zucman (2014) assume a value greater then one, Antras (2004) find values smaller than 1 and Chirinko (2008) defines a range of values between 0.4 and 0.6. Regarding the elasticity between land and capital, Ahlfeldt and McMillen (2014) find values close to 1. We assume the same value for the elasticity between capital and labor, and capital and land, in the housing sector and set it to $\sigma^h = 0.6$. We set the fraction of rich households to $m_r = 3\%$ to match the US share of households with income equal, or larger, than \$250,000 per year (US Census Bureau).

Finally, we set the population's growth rate to $n = 0\%$, and the parameter $\xi$ in the land policy equation (2), to 1, so that the amount of available land for construction adjusts proportionally to any, eventual, change in the population. Note that, by setting $\xi = 1$ we effectively shut down the effects of demography on the main variables. We leave for future work the analysis of a change in the population growth rate on wealth ratios, housing and land prices, and inequality.

Table 1: Model's Parameters

| | | |
|---|---|---|
| Preferences | | |
| Weight consumption young: | $\chi^y$ | 0.72 |
| Weight consumption old: | $\chi^o$ | 0.13 |
| Weight housing services: | $\chi^h$ | 0.15 |
| Elasticity of substitution preferences: | $\gamma$ | 0.50 |
| Interest rate: | $r$ | 0.28 |
| Technology | | |
| Housing depreciation (%): | $\delta$ | 0.20 |
| Capital share in housing: | $\alpha^h$ | 0.10 |
| Land share in housing: | $\eta^h$ | 0.50 |
| Capital share in manufacturing: | $\alpha^m$ | 0.35 |
| Elasticity of substitution housing: | $\sigma^h$ | 0.60 |
| Economy structure | | |
| Population growth rate (%): | $n$ | 0.00 |
| Fraction of rich households (%): | $m_r$ | 0.03 |
| Land policy rate: | $\xi$ | 1.00 |

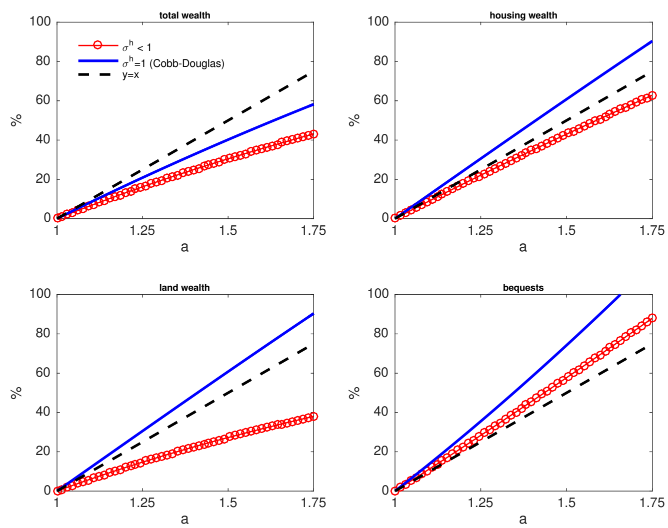
*Notes:* This table reports the parameters used to simulate the model for different steady-states corresponding to different values of the exogenous relative efficiency in manufacturing $a$. For robustness, we also run simulations for values of $\sigma^h = 1$ (*i.e.*, Cobb-Douglas technology). The real interest rate $r$ corresponds to a holding period equal to a generation.

We are interested in evaluating the long-run effects of productivity improvements and figures 5 and 6 summarize our main results. According to our estimates using O'Mahony and Timmer (2009)'s KLEMS data, on average, across the eight largest developed economies, relative labor efficiency in manufacturing increased approximately by 75 percent between 1970 to 2007[11]. Therefore, we plot the percentage changes in

[11]See the discussion in section 2.3 and the separate online appendix for details on the estimation of the relative labor efficiency, and a country breakdown of its evolution over the period 1970–2007, using O'Mahony and Timmer (2009)'s KLEMS data.

the steady-state values of the main variables of the model for different levels of the relative efficiency in manufacturing $a = 1, \ldots, 1.75$ with respect to their values when $a = 1$. Figure 5 plots the changes in total, housing and land wealth ratios as well as in bequests, our proxy for wealth inequality. We consider two different cases for the elasticity between inputs in the construction sector: in our baseline specification $\sigma^h$ is smaller than one and equal to 0.6 (red-dashed line); alternatively, we consider the case of $\sigma^h = 1$, *i.e.*, Cobb-Douglas (blue-solid line). All wealth ratios increase with the improvements in relative productivity, and the increase is stronger the larger the elasticity of substitution. For an increase of 75 percent of $a$ (*i.e.*, from 1 to 1.75), in the baseline specification of $\sigma^h < 1$, total wealth increases by about 40 percent, housing wealth by about 60 percent, land wealth by a 40 percent. Bequests, our measure of inequality, increase by approximately 90 percent. The effect of an increase in relative labor efficiency is stronger when technology in the housing sector is Cobb-Douglas. In the eight largest advanced economies, over the period 1970–2010, national wealth-to-income increased by approximately 60 percent, while housing wealth by 112 percent. Note that, despite the large increase in the simulations, housing wealth as a share of total wealth stays below 36 percent, in line with the average in the 8 largest economies in the period 1970–2010. Even though our stylized model can explain about half of the observed increases in wealth ratios, we stress that the simulation is not a proper quantitative exercise but, rather, an evaluation how far can a simple model go, without financial frictions, in explaining some of the long-run trends discussed in section 2. For example, the OLG model is quite stylized and not equipped to generate, for example, the large levels of wealth ratios we observe in the data. In fact, in our simulations wealth-to-income ratios are usually smaller than 1 and just above 1 for large values of $a$, while in the data they are a multiple of 1 in all countries. In addition, the strong increase in bequests implies an increase in inequality that is stronger than the one in the data without relying on alternative, established, explanations like skilled-biased technological change, the role of equity and financial markets, and less progressive tax systems. Also, our 2-type assumption implies that only a very small fraction of the population leaves any bequests and drives wealth inequality. Furthermore, while our numerical exercise compares different steady-states, the long-run trends in the data might combine different steady states and transitions within a given steady-state.

Figure 5: Wealth Ratios and Inequality

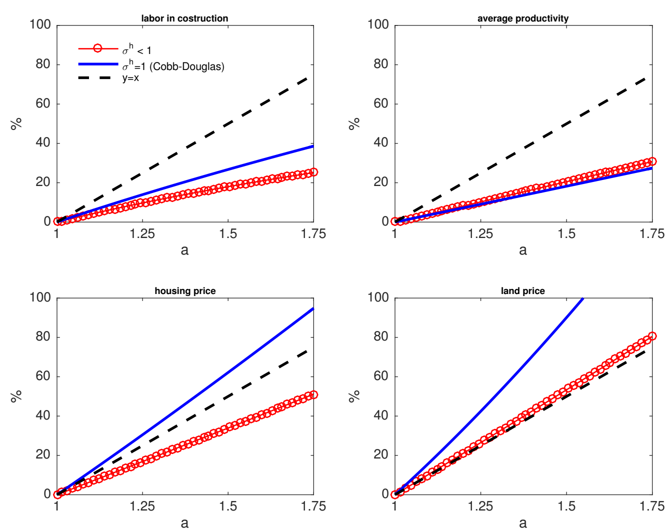



*Notes:* This figure plots the percentage changes in the steady-state values of total ($\beta$), housing ($\beta^h$) and land ($\beta^z$) wealth-to-income ratios and the level of bequests ($b$), for different values of the relative productivity sector $a = 1, \ldots, 1.75$, with respect to their value when $a = 1$. The red-dashed line is for the baseline value of the elasticity of substitution in the housing sector, $\sigma^h = 0.6$. The blue solid lines corresponds to Cobb-Douglas technology in the housing sector ($\sigma^h = 1$).

Figure 6 documents additional characteristics associated with a housing cost disease. First, we observe a re-allocation of labor toward the stagnant sector: the share of workers in construction ($\lambda$) increases by about 20 percent, with the increase stronger for larger values of $\sigma^h$. Despite the strong rise in $\lambda$, the share of workers in construction stays close to 5 percent percent in all cases. Second, average productivity increases less than proportionally exactly because of the shifts of workers to the less productive sector. Third, the housing price increases by a minimum of 50 percent ($\sigma^h < 1$) to a maximum of about 90 percent ($\sigma^h = 1$). Fourth, the land price responds strongly, and more than the housing price, to the increase in relative productivity in manufacturing:

it increases by about 80 percent for $\sigma^h < 1$ and by more than 100 percent for $\sigma^h = 1$. For the eight largest advanced economies, in the period 1970–2007, the mean increase in real housing prices was 53 percent and, in some countries like Italy and France, close to 100 percent. Therefore, our stylized model, with the caveats already discussed, has the capability of explaining a large fraction of the increase in housing and land prices observed since 1970.

Figure 6: Asset Prices and Re-allocation of Inputs




*Notes:* This figure plots the percentage changes in the steady-state values of the labor share in construction ($\lambda$), the average productivity ($y$), and the prices of houses ($q^h$) and land ($q^z$) for different values of the relative productivity sector $a = 1, \ldots, 1.75$, with respect to their value when $a = 1$. The red-dashed line is for the baseline value of the elasticity of substitution in the housing sector, $\sigma^h = 0.6$. The blue solid lines corresponds to Cobb-Douglas technology in the housing sector ($\sigma^h = 1$).

In this section, we presented results of numerical simulations of the model showing that it generates a strong *housing cost disease*. Despite the fact that manufacturing sector represents a small share to total valued added, changes in relative labor efficiency have the potential to generate large long-run increases in housing prices and land valuations. In the next section, we evaluates the implications of the *housing cost disease* in terms of welfare.

# 5 Welfare

In this section we address the housing cost disease problem from a welfare point of view and try to evaluate whether, according to our model, the fact that housing takes a large share of private wealth is undesirable.

Within a similar overlapping generations model, Deaton and Laroque (2001) find that the presence of a demand for housing in a growing economy generates a portfolio reallocation away from capital and towards housing, causing the accumulation of capital to fall short of the *Golden Rule* level, and they consider this as possible reason for confiscating property and giving it to consumers at no charge. Piketty (2014) advocates a tax on wealth based on the argument that a rising wealth to income ratio may lead to increasing inequality. Observing that housing takes a sizeable share of inter-generational bequests, Auerbach and Hasset (2015) note that reducing the tax benefits for owner-occupied housing in a progressive manner (or a deregulation in land use) may be more effective than a wealth tax in addressing the inequality problem. All of these conclusions must be taken with some caution. Regarding Deaton and Laroque's analysis, it should be noted that allocations departing from the Golden Rule are inconsistent with a social optimum only if we endorse a specific social welfare criterion, such as a weighted sum of all generations' utilities with rate of time preference equal to the population growth rate. In fact, any market allocation at which the rate of interest is larger than the population growth rate is Pareto optimal and, in these cases, reducing the value of the housing stock may have adverse effects on some generation's welfare. Conversely, when the real interest rate falls short of the population growth rate, a case that, in our model, can only occur with zero bequests, Pareto improvements can be obtained by decreasing investment in housing as well as in the capital stock. In other words, the crowding-out of capital induced by housing demand and the inter-generational transfers may, in fact, be desirable to avoid an over-accumulation of capital. With reference to inequality issue, there is a large literature showing that wealth taxes have ambiguous effects on the dispersion of wealth across heterogeneous individuals with altruist bequests motives (see Becker and Tomes (1979), Davies (1986), Benhabib et al. (2011), Zhu (2016)). To the best of our knowledge, none of these models have investigated the inequality issue when housing is part of total wealth. One basic intuition is that, leaving aside the question of whether estate taxes are effective in reducing inequality, the case for taxing housing is problematic, since housing is a consumption good as well as an asset, and a rise in after tax housing prices may reduce the poor households welfare.

Our goal is to evaluate the effect of a change in the reallocation of resources and prices induced by a rise in relative productivity on social welfare in a market economy, under an egalitarian and non-paternalistic criterion. This means that we consider a social welfare function biased against inequality (of individuals' utilities) and such that future generations' utilities for their own consumption are discounted with individuals' discount rates. There are a number of reasons why this question may be interesting. First, since rising housing prices generate more bequests and wealth inequality, the housing cost disease may be social welfare diminishing from an egalitarian perspective (although, due to heterogeneous discount rates, some inequality is compatible with an egalitarian planning optimum). Second, since, in a competitive equilibrium, poor households would like to force gifts from children and, therefore, leave no bequests because of one-sided altruism, raising the poor old individuals' wealth may improve social welfare. Then, a rise in housing prices may relax the non-negativity constraint on bequest values and generate the increase in old age consumption that the market is preventing under one-sided altruism. Third, a housing appreciation may decrease welfare as it makes housing less affordable.

As a first step in our discussion about the welfare implications of a rising labor efficiency in manufacturing, we compute the effect of an unanticipated rise in the level of the relative labor efficiency, $a = A^m/A^h$, at $t = 0$, for a constant labor efficiency in construction, $A_t^h = 1$ at a competitive equilibrium such that the poor-type households leave zero bequests and the rich-type leave positive bequests at all periods (*i.e.*, PBSS). Recall that, by forward iteration, the initial old type-$i$ household's utility is

$$V^{-1,i} = u(c_{-1}^{-1,i}, c_0^{-1,i}, h_0^{-1,i}) + \sum_{t=0}^{\infty} (\theta_i(1+n))^{t+1} u(c_t^{t,i}, c_{t+1}^{t,i}, h_{t+1}^i). \tag{47}$$

The impact of a rising $a$ on the above welfare function at a PBSS is evaluated by

$$\frac{\partial V^{-1,i}}{\partial a} = \frac{\mu^i}{a}\left[W + \left(\frac{1-\theta_i(1+n)}{\theta_i(1+n)}\right)\frac{q^z z^i}{(1+r)}\hat{q}_a^z + \left(\frac{(1-\delta)}{\theta_i(1+n)} - (1+r)\right)\frac{q^h h^i}{(1+r)}\hat{q}_a^h\right],$$

where

$$\mu^i = \frac{\theta_i(1+n)}{1-\theta_i(1+n)} u_1^i.$$

Notice that welfare is unambiguously enhanced by a land price appreciation, whereas the impact of a housing appreciation depends on subjective discount rates. Since $\theta_r = (1+r)^{-1}$,

$$\frac{(1-\delta)}{\theta_r(1+n)} - (1+r) = -(1+r)\left(\frac{\delta+n}{1+n}\right) < 0$$

and

$$\frac{(1-\delta)}{\theta_p(1+n)} - (1+r) \geq 0 \quad \Leftrightarrow \quad \theta_p < \frac{1-\delta}{(1+r)(1+n)}.$$

Therefore, if subjective discount rates are sufficiently similar, the housing price appreciation following an improvement in manufacturing productivity is welfare reducing for all households. In this case, the positive impact of a higher housing wealth on the initial old's utility is more than compensated by the negative impact on future generations' welfare due to the fact that housing becomes less affordable. The case $\theta_p \sim \theta_r$ appears to be significant if we assume that wealth inequality is mostly deriving from the initial distribution of assets across households, instead of the different degrees of altruism with respect to the offsprings. For example, assuming that the poor households own zero land and $\theta_p \to \theta_r$, we derive

$$a\frac{\partial V^{-1,p}}{\partial a} \to \mu^p q^h h^p \left(\frac{n+\delta}{1+n}\right)\left(\left(\frac{1+n}{\delta+n}\right)\frac{W}{q^h h^p} - \hat{q}_a^h\right). \tag{48}$$

In other words, an improvement in manufacturing productivity is welfare reducing for the poor households when the housing price elasticity with respect to $a$ is relatively large and the wage to housing wealth ratio is relatively small. The intuition is the following. As we know from the analysis in the previous sections, a rise in relative manufacturing productivity brings about a higher wage and a higher housing price. The former has a positive effect on poor households welfare, whereas the latter has two distinct and opposing effects. On the one hand, a higher housing price increases the old individual's wealth thereby increasing her utility. On the other, the higher housing price makes this good less affordable. The above analysis shows that the negative effect of a housing appreciation exceeds the former, and, if wages are sufficiently low relative to the value of housing, the negative welfare impact of the housing appreciation may overcome the positive impact due to higher wages.

What are the implications of the above findings on social welfare and government policies? In section B of the appendix we make some progress on this issue by considering the following egalitarian welfare function:

$$\mathcal{U} = \sum_i m_i \psi(V^{-1,i}), \tag{49}$$

where $\psi(.)$ is a positive and concave. Notice that, under strict concavity, the larger is the utility of the $i$-dynasty, the smaller is the increase in social welfare derived from raising this dynasty's welfare. This creates a relatively strong bias in favour of consumption and wealth equalization across households type. However, we should remind that, although the Planner is egalitarian, she takes into account the subjective discount factors, $\theta_i$, representing their degree of altruism with respect to the offsprings, in allocating resources. This implies that the poor (less altruistic) young households end up with a lower consumption than the rich (more altruistic) type asymptotically at the first best. In section B of appendix we provide a detailed analysis of the impact of $a$ on $\mathcal{U}$ and show that, if poor households own zero land (*i.e.*, $z^p = 0$), households discount rates, $\theta_i$, are sufficiently similar and inequality is relatively high, a rise in $a$ lowers the social welfare benefit of a higher value of $a$ relative to the first best when the right hand side of (48) is negative, *i.e.*, if housing appreciation is sufficiently strong and wages are sufficiently small at equilibrium.

# 6 Conclusions

In this paper we show that a Baumol's cost disease could complement alternative existing theories in explaining the increase in total and housing wealth-to-income ratios and wealth inequality that took place in the eight largest advanced economies in the last forty years. To show this, we have employed a simple life-cycle model with no financial frictions, two sectors (construction and manufacturing) and one-sided parental altruism. Key assumptions are that the construction sector is less capital intensive than manufacturing and housing demand sufficiently inelastic. Under these assumptions, a rise in labor efficiency in manufacturing produces a strong upward pressure on housing prices, a rise in the total and housing wealth-to-income ratios and a rise in bequests. The increase in housing valuations can possibly mitigate (relative to the First Best level) the beneficial effects of a rising productivity in manufacturing under an egalitarian welfare criterion when market allocations imply high enough consumption inequality and low enough heterogeneity in parental altruism. The empirical evidence supports our theoretical findings: there exists a positive link between relative productivity in the general economy, with respect to construction, and wealth ratios in the eight largest economies in the period 1970–2007.

# References


**Ahlfeldt, Gabriel M. and Daniel P. McMillen**, "New Estimates of the Elasticity of Substitution of Land for Capital," February 2014.

**Alvaredo, Facundo, Anthony B. Atkinson, Thomas Piketty, Emmanuel Saez, and Gabriel Zucman**, "The World Wealth and Income Database," 2016. http://www.wid.world.

**Antras, Pol**, "Is the US aggregate production function Cobb-Douglas? New estimates of the elasticity of substitution," *Contributions in Macroeconomics*, 2004, *4* (1).

**Auerbach, Alan J. and Kevin Hasset**, "Capita Taxation in the 21st Century," January 2015. NBER Working Paper 20871.

**Baumol, William**, "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis," *American Economic Review*, June 1967, *57* (3), 415–426.

**Becker, Gary S. and Nigel Tomes**, "An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility," *Journal of Political Economy*, December 1979, *87* (6), 1153–1189.

**Benhabib, Jess, Alberto Bisin, and Shenghao Zhu**, "The Distribution of Wealth and Fiscal Policy in Economies with Finitely Lived Agents," *Econometrica*, 2011, *79* (1), 123–157.

**_ , _ , and _** , "The wealth distribution in Bewely models with capital income risk," *Journal of economic theory*, 2015, *159*, 459–515.

**Bonnet, Odran, Pierre-Henri Bono, Guillaume Chapelle, and Etienne Wasmer**, "Le capital lodgement contribute-t-il aux inégalités?," April 2014. LIEPP Working Paper.

**Cagetti, Marco and Mariacristina De Nardi**, "Entrepreneurship, frictions, and wealth," *Journal of Political Economy*, 2006, *114* (5), 835–870.

**Castaneda, Ana, Javier Díaz-Gimènez, and José-Víctor Ríos-Rull**, "Accounting for U.S. earnings and wealth inequality," *Journal of Political Economy*, 2003, *111* (4), 818–857.

**Chirinko, Robert S**, "$\sigma$: The long and short of it," *Journal of Macroeconomics*, 2008, *30* (2), 671–686.

**Davies, James B.**, "Does redistribution reduce inequality?," *Journal of Labor Economics*, 1986, *4*, 538–569.

**Davis, Morris A. and Jonathan Heathcote**, "Housing and the Business Cycle," *International Economic Review*, 2005, *46* (3), 751–784.

**De Nardi, Mariacristina and Fang**, "Wealth Inequality, Family Background, and Estate Taxation," March 2015. mimeo.

**Deaton, Angus and Guy Laroque**, "Housing, Land Prices, and Growth," *Journal of Economic Growth*, June 2001, *6*, 87–105.

**Ermisch, John F., Jeanette Findlay, and Kenneth Gibb**, "The Price Elasticity of Housing Demand in Britain: Issues of Sample Selection," *Journal of Housing Economics*, 1996, *5*, 64–86.

**Favilukis, Jack, Sydney C. Ludvigson, and Stijn Van Nieuwerburgh**, "The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk Sharing in General Equilibrium," 2015. Journal of Political Economy (forthcoming).

**Gale, William G. and John Karl Scholz**, "Intergenerational transfers and the accumulation of wealth," *Journal of Economic Perspectives*, 1994, *8* (4), 145–160.

**Galor, Oded and Joseph Zeira**, "Income Distribution and Macroeconomics," *Review of Economic Studies*, January 1993, *60* (1), 35–52.

**Gourinchas, Pierre-Olivier and Jonathan A. Parker**, "Consumption over the life cycle," *Econometrica*, January 2002, *70* (1), 47–89.

**Grossmann, Volker and Thomas Steger**, "Das House-Kapital: A Theory of Wealth-to-Income Ratios," April 2016. CESifo Working Papers No. 5844.

**Hansen, Julia L., John P. Formby, and James W. Smith**, "The Income Elasticity of Demand for Housing: Evidence from Concentration Curves," *Journal of Urban Economics*, 1996, *39*, 173–192.

**Hanushek, Eric A. and John M. Quigley**, "What is the Price Elasticity of Housing Demand?," *Review of Economics and Statistics*, 1980, *62*, 449–454.

**Hendricks, Lutz**, "How important is discount rate heterogeneity for wealth inequality?," *Journal of Economic Dynamics and Control*, September 2007, *31* (9), 3042–3068.

**Hsieh, Chang-Tai and Enrico Moretti**, "Why Do Cities Matter? Local Growth and Aggregate Growth," April 2015. University of Chicago Working Paper.

**Iacoviello, Matteo**, "House prices, borrowing constraints, and monetary policy in the business cycle," *American Economic Review*, June 2005, *95* (3), 739–764.

_ **and Stefano Neri**, "Housing Market Spillovers: Evidence from an Estimated DSGE," *American Economic Journal: Macroeconomics*, April 2010, *2*, 125–164.

**Kahn, James A.**, "What drives housing prices?," *Federal Reserve Bank of New York Staff Reports*, 2008, (345).

**Knoll, Katharina, Moritz Schularick, and Thomas Michael Steger**, "No price like home: Global house prices, 1870-2012," 2014.

**Krusell, Per and Anthony A. Smith**, "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, October 1998, *106* (5), 867–896.

**Mayo, Stephen K.**, "Theory and Estimation in the Economics of Housing Demand," *Journal of Urban Economics*, 1981, *10*, 95–116.

**Moro, Alessio and Galo Nuno**, "Does total-factor productivity drive housing prices? A growth accounting exercise for four countries," *Economics Letters*, 2012, *115*, 221–224.

**Neels, Kevin**, "Specification Bias in Housing Production Functions," 1982. Rand note.

**Ngai, Rachel L. and Christopher Pissarides**, "Structural Change in a Multisector Model of Grwoth," *American Economic Review*, March 2007, *97* (1), 429–443.

**OECD**, *OECD Economic Surveys*, Paris: OECD Publishing, 2012.

**Ogaki, Masao and Carmen M Reinhart**, "Measuring intertemporal substitution: The role of durable goods," *Journal of Political Economy*, 1998, *106* (5), 1078–1098.

**O'Mahony, Mary and Marcel P. Timmer**, "Output, Input and Productivity Measures at the Industry Level: the EU KLEMS Database," *Economic Journal*, 2009, *119* (538), F374–F403.

**Piketty, Thomas**, *Capital in the Twenty-First Century*, Cambridge, Massachusetts: Belknap Press - Harvard, 2014.

_ **and Emmanuel Saez**, "Inequality in the long run," *Science*, May 2014, *344* (6186).

_ **and Gabriel Zucman**, "Capital is Back: Wealth Income Ratios in Rich Countries 1700-2010," *Quarterly Journal of Economics*, 2014, *129* (3), 1255–1311.

**Quadrini, Vincenzo**, "Entrepreneurship, saving, and social mobility," *Review of Economic Dynamics*, 2000, *3* (1), 1–40.

**Rognlie, Matthew**, "A Note on Piketty and diminishing returns to capital," 2014. mimeo.

**Summers, Larry**, "The Inequality Puzzle," Democracy A Journal of Ideas 2014. Summer.

**Valentinyi, Akos and Berthed Herrendorf**, "Measuring Factor Income Shares at the Sector Level," *Review of Economics Dynamics*, October 2008, *11*, 820–835.

**Weil, David N.**, "Capital and Wealth in the 21st Century," 2015. NBER Working Paper 20919.

**Zhu, Shenghao**, "A Becker-Tomes Model with Investment Risk," January 2016. mimeo.

# Appendix (for publication)[†]

## A Comparative Statics

### A.I Labor Allocation and Asset prices

Since the land policy (19) implies $z = \xi/\lambda$, equation (24) implies that the share of labor in construction is determined implicitly by $v$ (and $a$) through the following equation

$$\frac{\lambda f^h(k^h(r,a),\xi/\lambda)}{f_k^h(k^h(r,a),\xi/\lambda)} = \left(\frac{\delta+n}{1+r}\right) v. \tag{A1}$$

It is possible to show that the left hand side of (A1) is increasing in $\lambda$, so that following proposition holds.

**Proposition 3.** *For all $v \in [0, v^m(a)]$, with*

$$v^m(a) = \frac{f_k^h(k^h(r,a),\xi)}{f^h(k^h(r,a),\xi)} \left(\frac{1+r}{\delta+n}\right),$$

*there exists a value $\lambda(v,a) \in [0,1]$ solving equation (A1), such that $\lambda_v(v,a) > 0$, $\lambda_a(v,a) < 0$ and $\lambda(0,a) = 0$, $\lambda(v^m(a),a) = 1$.*

*Proof.* Letting

$$\phi(\lambda,a) = \lambda f^h(k^h(r,a),\xi/\lambda)/f_k^h(k^h(r,a),\xi/\lambda),$$

direct computation shows that

$$\phi_\lambda = (k^h/\mathcal{S}_k^h)\left(1 - \mathcal{S}_z^h + (\mathcal{S}_k^h/\sigma^h)(k^h/\mathcal{S}_k^h)^{\frac{\sigma^h-1}{\sigma^h}}\right), \tag{A2}$$

$$\phi_a = \lambda(k^h/a)(\sigma^h + (1-\mathcal{S}_k^h)/\mathcal{S}_k^h), \tag{A3}$$

so that

$$\lambda_v(v,a) = \frac{(\delta+n)}{(1+r)\phi_\lambda} > 0, \qquad \lambda_a(v,a) = -\frac{\phi_a}{\phi_\lambda} < 0.$$

□

Quite intuitively, a rise in housing wealth generates a reallocation of labor toward the construction sector and a rise in $a$ a reallocation of labor away from this sector.

[†] This appendix is intended for publication. An additional online appendix, not for publication, is available on our websites and contains details on the data and further robustness checks.

Note that these are only partial effects: in equilibrium, housing wealth, $v$, is affected by $a$, so that a rise in productivity in the manufacturing sector may shift labor to the construction sector if this has a positive and large enough effect on $v$.

By the mapping $\lambda = \lambda(v, a)$ and the land policy (19), we derive the steady state asset prices defined in (25), (26), (27) as functions of $(v, a)$ as well. Observe that the effect of a higher housing wealth on asset prices is unambiguously positive. In fact, since $\lambda_v(v, a) > 0$, a rise in housing wealth generates a larger labor share in construction, reduces the land-to-labor ratio in the construction sector and, then, it reduces the marginal product of capital and it increases the marginal product of land. Since $r$ is pinned down by $\theta_r$, all asset prices increase. On the other hand, the impact of $a$ on asset prices for given $v$ is ambiguous. When land is totally unproductive, $q^h$ is independent of $v$ and, by differentiation of $q^h$ with respect to $a$, we derive $\partial q^h/\partial a > 0$. Hence, similarly to the Baumol's prediction, a rise in productivity in manufacturing would generate a rise in the relative price of the output of the stagnant sector. In more general cases, $a$ affects $\lambda$ (for any given $v$) and the latter affects the land-labor ratio, $z$. In particular, since $z = \xi/\lambda(v, a)$, $\lambda_a < 0$, and the CES specification implies $f_{h,k,z} > 0$, we have that $q^h$ and $q^z$ are increasing in $a$ for given $z$ and decreasing in $z$ for given $a$. Hence, for given $v$, a rise in $a$ has two effects on asset prices. The first is a positive direct effect because of a higher capital labor ratio in the construction sector. The second is a negative indirect effect through a higher land-labor ratio induced by a lower share of labor in this sector. We can provide an analytical representation of these effects in terms of factor shares. In particular, the partial elasticities of $q^h(v, a)$ and $q^z(v, a)$ are

$$\hat{q}_v^h = \frac{\mathcal{S}_z^h (q^h y^h/(1+r))^{\frac{\sigma_h - 1}{\sigma_h}}}{\sigma^h (1 - \mathcal{S}_z^h) + \mathcal{S}_z^h (q^h y^h/(1+r))^{\frac{\sigma_h - 1}{\sigma_h}}}, \qquad \hat{q}_v^z = \frac{1 - (1 - \sigma^h)\hat{q}_v^h}{\sigma^h} \tag{A4}$$

and

$$\hat{q}_a^h = (1 - \mathcal{S}_k^h) - ((1 - \mathcal{S}_k^h) + \sigma^h \mathcal{S}_k^h)\hat{q}_v^h, \qquad \hat{q}_a^z = -\left(\frac{1 - \sigma^h}{\sigma^h}\right)\hat{q}_a^h. \tag{A5}$$

The basic insights are that the partial elasticities of $q^h$ and $q^z$ with respect to $v$ are positive, with $\hat{q}_v^h$ in $[0, 1]$ and $\hat{q}_v^z \geq \hat{q}_v^h$, that $q^z$ is increasing in $a$ if and only if $\sigma^h \geq 1$ and that

$$\hat{q}_a^h \geq 0 \qquad \Leftrightarrow \qquad \hat{q}_v^h \leq \frac{(1 - \mathcal{S}_k^h)}{(1 - \mathcal{S}_k^h) + \sigma^h \mathcal{S}_k^h}.$$

Note that the condition above holds when $\mathcal{S}_z^h$ is not too large or $\mathcal{S}_k^h$ is sufficiently close to zero. However, more general conditions may also generate a positive relation between $q^h$ and $a$. In the specific case of a Cobb-Douglas production function in the construction sector,

$$\hat{q}^h_v = \mathcal{S}^h_z, \qquad \hat{q}^z_v = 1, \qquad \hat{q}^h_a = \mathcal{S}^h_l, \qquad \hat{q}^z_a = 0. \tag{A6}$$

## A.II Upper Bound on $\Delta$ with Zero Bequests

**Proposition 4.** *Assume that $b^r = 0$. Then, $\Delta < 1$ at equilibrium.*

*Proof.* Since $\Delta = (\delta + n)(ak^m - k^h)/q^h y^h$, we have

$$1 - \Delta = \frac{q^h y^h - (\delta + n)(ak^m - k^h)}{q^h y^h}. \tag{A7}$$

Since $v = \lambda q^h y^h/(\delta + n)$, $G^s(b, v, a) = 0$ implies

$$\lambda(q^h y^h - (\delta + n)(ak^m - k^h)) = \frac{\delta + n}{1 + n}(s - (1 + n)(ak^m + q^z \xi)).$$

Now let $1 - \Delta + q^z \xi/v < 0$. Then, $\lambda \in [0, 1]$ if and only if $s < (1 + n)(ak^m + q^z \xi)$ and

$$s > \left(\frac{1 + n}{\delta + n}\right) q^h y^h + (1 + n)(k^h + q^z \xi).$$

Since $\delta \leq 1$, the above requires

$$s > q^h y^h \geq W.$$

□

## A.III Proof of proposition 2

First of all, we prove the following lemma.

**Lemma 1.** *For any given $(\pi, aw)$, and $b^i > 0$,*

$$\partial s^i(\pi, aw, b^i)/\partial b^i > \partial q^h h^i(\pi, aw, b^i)/\partial b^i > 0.$$

*Proof.* From the individuals' budget constraints, and for any demand function, $(c^{y,i}, c^{o,i}, h^i)$, we have

$$s^i = W + b^i - c^{y,i}(.), \qquad c^{y,i}_I(.) + c^{o,i}_I(.)/(1 + r) + \pi h^i_I(.) = 1,$$

where the subscript $I$ on each demand function denotes the partial derivative with respect to $I^i$. Then,

$$\frac{\partial s^r(.)}{\partial b} = 1 - \left(\frac{r-n}{1+r}\right) c_I^{y,r}(.) = \left(\frac{r-n}{1+r}\right) c_I^{y,r}(.) + \frac{c_I^{o,r}(.)}{1+r} + \pi h_I^r(.)$$

and

$$\frac{\partial q^h h^r(.)}{\partial b} = \left(\frac{r-n}{1+r}\right) q^h h_I^r.$$

Since $q^h = (1+r)\pi/(r+\delta)$,

$$\frac{\partial q^h h^r(.)}{\partial b} = \left(\frac{r-n}{\delta+r}\right) \pi h^r < \pi h^r < \frac{\partial s^r(.)}{\partial b}.$$

Since $\delta + n > 0$, we get the proposition. $\square$

We prove now that the whole equilibrium system at a PBSS reduces to two equations and two unknowns. The two equations are the market clearing condition in the housing market and the capital market equilibrium; the two unknowns are the rich households' bequests, $b$, and the value of housing wealth, $v$. This is accomplished by using (24) to derive a mapping

$$(v, a) \to (\lambda, k, q^h, q^z, \pi)$$

and using the market clearing conditions for housing and capital markets (21)-(22) at steady state to derive the excess demand for housing wealth and the excess supply of saving (over investment) as

$$G^d(b, v, a) \equiv q^h(v, a) h^d(\pi(v, a), aw, b) - v, \tag{A8}$$

$$G^s(b, v, a) \equiv \frac{1}{1+n} s(\pi(v, a), aw, b) - k(v, a) - v - q^z(v, a)\xi. \tag{A9}$$

Then, the steady state specification of the equilibrium conditions (21)-(22) define the reduced form equilibrium steady state conditions for any given $a$ as

$$G^d(b, v, a) = 0, \tag{A10}$$

$$G^s(b, v, a) = 0, \tag{A11}$$

and a PBSS is a positive pair, $(b^*(a), v^*(a))$, such that

$$0 = G^d(b^*(a), v^*(a), a) = G^s(b^*(a), v^*(a), a). \tag{A12}$$

Letting $v^d(b, a)$ and $v^s(b, a)$ be the solutions for $v$ to (A10) and (A11), respectively, for a given pair $(b, a)$, we say that $v^d$ is the demand and $v^s$ the supply of housing wealth. Intuitively, $v^d$ is the households' real expenditure for the stock of available housing and $v^s$ defines the amount of housing wealth that is consistent with a capital market equilibrium, *i.e.*, with the available amount of savings, business capital and land value. It turns out that, if the economy is *unit elastic*, *i.e.*, $\sigma^h = \gamma = 1$, and (44) is verified, the demand and supply schedules, *i.e.*, $v^d(b, a)$ and $v^s(b, a)$ in the space $(b, v)$, are well defined, have a unique intersection at $b^*(a)$, where $v^s$ is steeper than $v^d$ (*i.e.*, $v_b^s > v_b^d$). The case of a demand schedule less steep than the supply schedule, which we refer to as a *regular intersection*, follows from the assumption (44) and the property (34), *i.e.*, the fact that a rise in bequests has a greater impact on households' saving than on their expenditure on housing. The unit-elastic economy appears, then, to be a natural benchmark because, coupled with the capital intensity assumption 4, is well behaved (*i.e.*, it allows for meaningful comparative statics results) and, as we show in proposition 2, it implies that both $b^*(a)$ and $v^*(a)$ are unit-elastic with respect to $a$ and that changes in $a$ are neutral with respect to the shares of labor across sectors as well as on the wealth-to-income ratios.

Now recall that, by (35), $\gamma = 1$ implies $1 + \hat{h}_\pi^d = (1 - \gamma)(1 - \phi^h) = \hat{s}_\pi = 0$ and, because $\sigma^h = 1$, $\Delta_v = \mathcal{S}_{z,v}^h = 0$. Then, recalling (A8), (A9), it follows that, when $\gamma = \sigma^h = 1$,

$$G_v^d = -1, \qquad G_v^s = -(1 - \Delta + (\delta + n)\mathcal{S}_z^h / r),$$

and, hence, (44) and $\sigma^h = \gamma = 1$ imply

$$-G_v^d = 1 > -G_v^s > 0. \tag{A13}$$

By lemma 1, we have $G_b^s > G_b^d > 0$, so that, by (A13), when $\sigma^h = \gamma = 1$,

$$v_b^s = \frac{G_b^s}{-G_v^s} > \frac{G_b^d}{-G_v^d} = v_b^d.$$

Then, when $\gamma = \sigma^h = 1$ (unit-elastic economy), the housing supply schedule is steeper than the housing demand. We identify this case with a "well behaved economy" and, from now on, we impose assumption 4 together with the assumption that $\gamma$ is sufficiently close to one, so that the above inequality is verified. Defining the partial elasticities of housing demand as

$$\hat{v}_a^j = v_a^j(b, a)a/v, \qquad \hat{v}_b^j = v_b^j(b, a)b/v,$$

for $j = d, s$, we obtain

$$\hat{b}_a = \frac{\hat{v}_a^d - \hat{v}_a^s}{\hat{v}_b^s - \hat{v}_b^d}. \tag{A14}$$

Since demand functions are homothetic, we get

$$aG_a^j + bG_b^j = \Sigma^j, \qquad \text{for } j = d, s,$$

where, by the assumption $\sigma^h = 1$,

$$\Sigma^d = v(\mathcal{S}_l^h(1-\gamma)(1-\phi^h) + 1), \tag{A15}$$

$$\Sigma^s = v(1 - \Delta + (\delta + n)\mathcal{S}_z^h/r) - s\hat{s}_\pi \mathcal{S}_l^h. \tag{A16}$$

Using the above in (A14) we get

$$\hat{b}_a^* = 1 + \Gamma, \qquad \hat{v}_a^* = 1 + \frac{(1-\mathcal{S}_k^h)(1-\gamma)(1-\phi^h)}{1-\mathcal{S}_z^h(1-\gamma)(1-\phi^h)} + \hat{v}_b^d\Gamma, \tag{A17}$$

where

$$\Gamma = \frac{1}{\hat{v}_b^s - \hat{v}_b^d}\left(\frac{\Sigma^d}{-vG_v^d} - \frac{\Sigma^s}{-vG_v^s}\right).$$

By rearranging terms,

$$\Gamma = \frac{(1-\gamma)}{(\hat{v}_b^s - \hat{v}_b^d)G_v^d G_v^s} M,$$

where

$$M = (1 - \Delta + (\delta+n)\mathcal{S}_z^h/r)v(\mathcal{S}_l^h + \mathcal{S}_z^h) + \frac{\phi^h c^y}{1+n}(\mathcal{S}_l^h - \mathcal{S}_z^h - 2(1-\gamma)(1-\phi^h)\mathcal{S}_l^h\mathcal{S}_z^h).$$

Then, by lemma 1, assumption 4 and since $\gamma$ is close to one, it is $\Gamma > 0$ and

$$\hat{b}_a^* \geq 1, \qquad \hat{v}_a^* \geq 1 + \eta$$

where

$$\eta \equiv \frac{(1-\mathcal{S}_k^h)(1-\gamma)(1-\phi^h)}{1-\mathcal{S}_z^h(1-\gamma)(1-\phi^h)}.$$

It follows that $\hat{b}_a^* = \hat{v}_a^* = 1$ if and only if $\gamma = 1$. Observe that the appropriate range of $\gamma$ for which $M > 0$ is verified under assumption 4 is given by

$$1 > \gamma > 1 - \frac{1}{1-\phi^h}\left(\frac{\mathcal{S}_l^h - \mathcal{S}_z^h}{2\mathcal{S}_l^h\mathcal{S}_z^h}\right), \tag{A18}$$

Using (41), (42), (A2), (A3), (A6), the equilibrium elasticities under the assumption $\sigma^h = 1$ are given by

$$\begin{aligned}
\hat{y}_a^* &= 1 - \beta^h((1+r)\Delta - (\delta+n)\mathcal{S}_z^h)(\hat{v}_a^* - 1), && \text{(A19)}\\
\hat{\beta}_a^{h,*} &= (1 + \beta^h((1+r)\Delta - (\delta+n)\mathcal{S}_z^h))(\hat{v}_a^* - 1), && \text{(A20)}\\
\hat{\beta}_a^* &= \frac{\beta^{h,*}}{\beta}\left[\beta((1+r)\Delta - (\delta+n)\mathcal{S}_z^h) + 1 - \Delta + (\delta+n)\frac{\mathcal{S}_z^h}{r}\right](\hat{v}_a^* - 1), && \text{(A21)}\\
\hat{\lambda}_a^* &= (\hat{v}_a^* - 1)/(1 - \mathcal{S}_z^h), && \text{(A22)}\\
\hat{q}_a^{h,*} &= \mathcal{S}_l^h + \mathcal{S}_z^h \hat{v}_a^*, && \text{(A23)}\\
\hat{q}_a^{z,*} &= \hat{v}_a^*. && \text{(A24)}
\end{aligned}$$

Then, for the unit-elastic economy,

$$\hat{y}_a^* = 1, \quad \hat{\beta}_a^* = \hat{\beta}_a^{h,*} = \hat{\lambda}_a^* = 0, \quad \hat{q}_a^{h,*} = 1 - \mathcal{S}_k^h, \quad \hat{q}_a^{z,*} = 1 - \mathcal{S}_z^h.$$

If, on the other hand, $\gamma < 1$, we have $\hat{v}_a^* - 1 > \eta > 0$, and then, by inspection of (A19)-(A22), wealth-to-income ratios and the share of labor employed in construction increase with $a$, whereas average labor productivity falls or increases by less than proportionally with $a$. Regarding the price elasticities, in this case we get the lower bounds in (46).

# B Welfare

The functional (47) is assumed to be well defined and finite. By exploiting the envelope theorem, indirect effects on individuals decisions can be shown to be irrelevant, and we derive

$$\frac{\partial V^{-1,i}}{\partial a} = u_{2,-1}^i\left((1-\delta)h_0^i\frac{\partial q_0^h}{\partial a} + z_0^i\frac{\partial q_0^h}{\partial a}\right)$$
$$+\sum_{t=0}^{\infty}\rho_t^i\left(\frac{\partial W_t}{\partial a} + \frac{c_{t+1}^{t,i}}{(1+r_{t+1})^2}\frac{\partial r_{t+1}}{\partial a} - h_{t+1}^i\frac{\partial \pi_t}{\partial a}\right), \qquad \text{(A25)}$$

where $z_0^i$ is the individuals' initial endowment of land and

$$\rho_t^i \equiv (\theta_i(1+n))^{t+1}u_{1,t}^i,$$

are the time-$t$ subjective prices of households of type $i \in \{p, r\}$.

Now consider a steady state, where $c_t^{t,i} = c^{y,i}$, $c_t^{t-1,i} = c^{o,i}$ for all $t \geq 0$ are the stationary young and old age consumptions of the two types, and define

$$\mu^i \equiv \sum_{t=0}^{\infty} \rho_t^i = \frac{\theta_i(1+n)}{1-\theta_i(1+n)} u_1^i.$$

Then, we can decompose the impact of a rising $a$ on households (dynastic) welfare into two parts: the effect arising from a change in the wage rate, $W$, and the effect arising from a change in asset prices. In particular, recall that, at a PBSS, $r_t = r = 1/\theta_r - 1$ for all $t \geq 0$ and $W_t = aw(r)$. Then,

$$\frac{\partial V^{-1,i}}{\partial a} = \frac{\mu^i}{a}\left[W + \left(\frac{1-\theta_i(1+n)}{\theta_i(1+n)}\right)\frac{q^z z^i}{(1+r)}\hat{q}_a^z + \left(\frac{(1-\delta)}{\theta_i(1+n)} - (1+r)\right)\frac{q^h h^i}{(1+r)}\hat{q}_a^h\right].$$

Consider now the social welfare function in (49). It is straightforward to verify that, for some given amount of land and public spending, a competitive equilibrium corresponds to a social optimum (conditional on the given amount of land and social spending) provided that the non-negativity constraint on bequests is not binding and

$$\psi'(V^{-1,i})\rho_t^i = \frac{(1+n)^t}{\Pi_{j=1}^t(1+r_j)}\mu_0 \tag{A26}$$

for all $i \in \mathcal{I}$ and some scalar $\mu_0 > 0$. In other words, sub-optimality may only occur because the Planner would not generate wasteful spending and, for some households, she may provide negative bequests, which is impossible in the market economy. Regarding bequests, we have

$$u_{2,t}^i = \theta_i u_{1,t+1}^i$$

for all $i$ at a planning optimum. Notice, also, that, although the Planner is egalitarian, she takes into account the subjective discount factors, $\theta_i$, representing their degree of altruism with respect to the offsprings, in allocating resources. Using (A26) and the remaining first order characterization of the planning optimum for a CES specification of individuals' preferences, one can easily show that, under the assumption $\theta_r > \theta_p$,

$$\lim_{t\to\infty} \frac{c_t^{t,p}}{c_t^{t,r}} = 0,$$

*i.e.*, the Planner's allocation is such that the poor (less altruistic) type young households end up with a lower consumption than the rich (more altruistic) type.

Now we consider the effect on the Planner's welfare function, $\mathcal{U}$, of a once and for all rise in $a$ at a PBSS, assuming that poor households own zero land (*i.e.*, $z^p = 0$). By exploiting the equilibrium conditions (*i.e.*, resource feasibility, first order conditions for individual optimality at equilibrium and budget constraints) and letting $\eta^i \equiv \psi'(V^{i,-1})$, we derive

$$a\frac{\partial \mathcal{U}}{\partial a} = \eta^r \mu^r \left(W(1-\lambda) - a\frac{\partial g}{\partial a}\right) + m_p \Psi, \tag{A27}$$

where

$$\Psi = (\eta^p \mu^p - \eta^r \mu^r)W + \hat{q}_a^h \frac{q^h h^p}{1+r}\left(\left(\frac{\delta+n}{1+n}\right)\left(\frac{\eta^r \mu^r}{\theta_r} - \frac{\eta^p \mu^p}{\theta_p}\right) + (1+r)\left(\frac{\theta_r - \theta_p}{\theta_p}\right)\right).$$

Since a First Best allocation is such that $\eta^p \mu^p = \eta^r \mu^r$, the first summation on the right hand side of (A27) represents the *undistorted* component of the welfare effect of a rising $a$ conditional on the wasteful government spending of the revenues from land sale, whereas $\Psi$ represents two possible distortions: the first one arising from the possibility that consumption is not allocated as dictated by the Planner across individuals of the same generation, and the second from the fact that the poor-type households are unable to leave positive bequests to their offsprings. Observe that, at equilibrium, the sign of $\eta^p \mu^p - \eta^r \mu^r$ is ambiguous, since it depends on the differences between consumptions across the two type of households (in the same age and time), the dynastic-specific discount factors and utilities. If the discount factors are very similar and the rich households have a much larger consumption compared with the poor household, we have $\eta^p \mu^p - \eta^r \mu^r > 0$. We say that the impact of $a$ on social welfare falls short of the first best level net of wasteful government spending if $\Psi < 0$.

To evaluate the sign of $\Psi$ we concentrate on a case that appears to be particularly relevant in our model. Namely, we assume CES utility function (as in the previous sections) and we evaluate $\Psi$ for $\theta_p$ converging to $\theta_r$. With a CES representation of preferences, utility is linear in households' income and the marginal utilities of age-contingent consumptions and housing are identical across households at steady states, so that $u_1^p = u_1^r$. Then, due to the concavity of $\psi$, when subjective discount rates are sufficiently similar across households, we get $\mu^p > \mu^r$ (since $b^r > b^p = 0$ implies $V^{-1,p} < V^{-1,r}$) and, then,

$$\Psi < 0 \qquad \Leftrightarrow \qquad \hat{q}_a^h > \left(\frac{1+n}{\delta+n}\right)\frac{W}{q^h h^p}.$$